Intelligent Controller Design

for

Configurable Control Systems

A *thesis submitted*

in partial fulfillment of the requirements

for the degree of

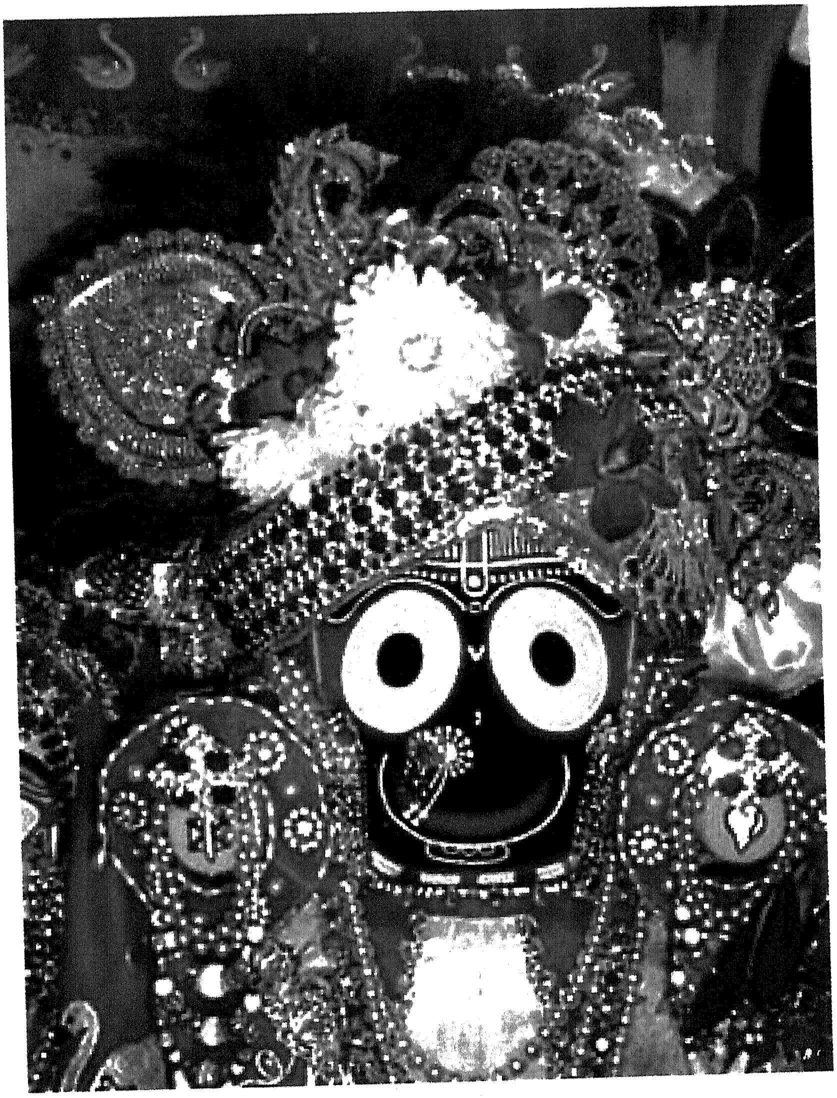
Master of Technology

by

Gopal Krushna Das.

to the
DEPARTMENT OF ELECTRICAL ENGINEERING,
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR
U.P., KANPUR, INDIA.
JULY 2004

Dedicated at the lotus feet of.....



CERTIFICATE

This is to be certified that the work entitled **"Intelligent Controller Design for Configurable Control System"** by Gopal Krushna Das, Roll No-Y3104032, has been carried out under my supervision for the partial fulfillment of M. Tech. Degree in the department of Electrical Engineering, IIT Kanpur and this work has not been submitted elsewhere for any other degree.

L Behera

12th July 2005

Dr. Laxmidhar Behera
Dept. of Electrical Engineering,
Indian Institute of Technology, Kanpur.

POST GRADUATE OFFICE
THESIS SUBMITTED
ON 12/7/05
I.I.T., KANPUR-208 016

Acknowledgement

I would like to give my heart-felt gratitude to **Dr. Laxmidhar Behera** for providing me the necessary guidance and inspiration to carry out this work. He gave me enough freedom to make experiment with my ideas and always there to help me out whenever i got into troubles. this work won't have been possible without his constant support and motivation.

I would like to thank **Mr C.K. Pithawa** and **Mr Uday Vaidya** who gave me all the data for making the modeling of nuclear steam generator.

I would like to thanks my seniors **Swagat Kumar, Awhan Pattnaik, Indrani Kar and Anjan Ray** for their constructive suggestions and invaluable help during my work.

I would like to thanks my colleagues **Prem Kumar, K N Srinu and Ravindra Kumar Sharma** with them I always discuss the problem before further progress.

I would like to thanks my friends in IIT Kanpur for their good wishes and support in various ways.

Finally, I would like to extend my thanks to whoever made the nice environment in IIT Kanpur for making good research.

Abstract

Fuzzy Logic Controllers (FLCs) using Genetic Algorithm have been designed for four different dynamical systems. The parameters of fuzzy logic controller have been determined optimally by using genetic algorithm with multi objective cost function.

1. Poor water level control of U tube steam generator (UTSG) in nuclear power plant may lead to frequent nuclear shutdown. UTSG of pressurized water reactor (PWR) has a time varying dynamics. FLC has been designed for this system and its performance is compared with LQR controller. It is found that its performance is as comparable with LQR controller.

2. Control of a U tube steam generator is possible if a proper mathematical model can be derived. A U tube steam generator has been modeled for analyzing its behavior. The PD, PI and fuzzy logic controllers have been implemented for controlling the water level.

3. FLC controller has been implemented on a single link manipulator and its performance has been tested with linearized and PD controllers. It is found that FLC performs better than PD Controller and is well compared with linearized control schemes.

4. FLC has been used to balance a reaction wheel pendulum, which is an under actuated system.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

1.1 Introduction

All natural systems are inherently nonlinear. The properties of nonlinear systems are quite different from those of linear systems. For example, nonlinear systems exhibit properties like limit cycles, chaos, bifurcation etc. Moreover its stability depends not only on the initial condition but also on the control input. Linear system theory is very well developed and numerous tools are available for its analysis. In nonlinear systems, the tools available are limited and they are problem specific. Various linear system techniques have been used to analyze and control the linearized model of nonlinear plants. Recently, a number of other tools belonging to class of "learning systems" have been developed for designing high performance controllers. This later class of tools include Fuzzy logic, neural networks, evolutionary algorithms etc.

This thesis consists of two parts. In first part, we discuss about the modeling and control of a nuclear boiler and in the other, we discuss about various controllers for nonlinear mechanical systems.

1.2 Level Control of Steam Generators

Nuclear power is identified as one of the major electric power source in future. These power plants are very sensitive to any disturbance (mechanical and electrical) due to various technical and safety reasons. U tube steam generator is a major part of nuclear power plant. It forms a vital link between the heat source (the reactor) and the heat sink (the turbo generator system). The prediction of this behavior requires the knowledge of non-linear dynamical models.

Controlling the water level in a large PWR plant is of major importance to ensure sufficient cooling of reactor, good performance of steam separators and dryers. Simultaneously it is required to eliminate all risks of hydrodynamic instability [1]. As a result, a good control system is of paramount importance for the proper functioning of the nuclear plant.

Since the actual plant is nonlinear and complex, most of the time, linearized model is used for designing controllers. For linearized plants, controllers like LQR and state-feedback control have been used extensively in the literature [2] [3]. Keeping the complex nature of boiler dynamics, a number of techniques based on soft computing approaches like "Fuzzy Logic" have been developed for designing controllers. [4] [5]. Fuzzy logic controllers have been shown to improve the performance of existing conventional controllers.

1.3 Control of nonlinear mechanical systems

Nonlinear dynamical system like single link manipulator, reaction wheel pendulum are used as a bench mark problem for control system. Some systems are under actuated

which are more difficult to control. For nonlinear plant, a number of different controllers have been used in the literature [6] [7] [8]. Different non linear control schemes like adaptive control, fuzzy logic control, and sliding mode control have been described in the literature. In the present work fuzzy logic control is used to control three non linear control systems.

1.4 Contribution

The performance of FLC has been compared with those of conventional controllers for different class of nonlinear systems. The nonlinear systems studied here, include single link manipulator, reaction wheel pendulum and steam generator. The fuzzy parameters have been optimized using genetic algorithm.

Detailed modeling of a U tube steam generator has been carried out based on the technical data provided by BARC.

A fuzzy logic controller has been designed for water level control of U tube steam generator in a nuclear power plant and its performance is compared with those of conventional controllers.

All the simulation works have been carried out in C++ language.

1.5 Thesis Organization

The thesis is organized as follows.

Chapter 2 describes briefly about Fuzzy Logic and Genetic Algorithm.

In **Chapter 3**, LQR and Fuzzy Logic controllers are used for level control of U tube steam generator. Here we use a linearized model of the actual plant.

The modeling of Nuclear steam generator has been carried out in **Chapter-4**. Three different controllers have been implemented on this model.

In **Chapter 5**, performance of different controllers are compared for a single link manipulator problem.

The balancing problem of a reaction wheel pendulum is analyzed in **Chapter 6**. Performance of different controllers are compared for balancing task.

The thesis is summarized and its future scope is discussed in **Chapter 7**.

The overview of different controllers and formulae related to Boiler dynamics have been provided in the **Appendix**.

***

Chapter 2

Introduction to Fuzzy Logic and Genetic Algorithm

2.1 Introduction to Fuzzy Logic

In the real world, information is often ambiguous or imprecise. When we state that it is warm today, the context is necessary to approximate the temperature. A warm day in January may be -5 degrees celsius, but a warm day in August my be 35 degrees. After a long spell of frigid days, we may call a milder but still chilly day relatively warm. Human reasoning filters and interprets information in order to arrive at conclusions or to dismiss it as inconclusive. Although machines cannot yet handle imprecise information in the same ways that humans do, computer programs with fuzzy logic are becoming quite useful when the sheer volume of tasks defies human analysis and action.

An organized method for dealing with imprecise data is called fuzzy logic. The data are considered as fuzzy sets. For traditional sets, there is no other case than true or false. Fuzzy sets allow partial membership. Fuzzy Logic is basically a multi valued logic that allows intermediate values to be defined between conventional evaluations like yes or no, true or false, black or white etc. Notions like rather warm or pretty cold can be formulated mathematically and processed with the computer. In this way an

attempt is made to apply a more human-like way of thinking in the programming in the computers.

2.2 Fuzzy Logic - Its Origin

The concept of fuzzy logic (FLC) was conceived by Lotfi Zadeh, a professor at the University of California at Berkley, and presented not as a control methodology, but as a way of processing data by allowing partial set membership rather than crisp set membership or non-membership [4]. Fuzzy logic was not applied to control systems before 70's due to insufficient micro computer capability prior to that time. Professor Zadeh reasoned that people do not require precise, numerical information input, and yet they are capable of highly adaptive control.

2.3 How Fuzzy Logic is Different from Conventional Control Methods?

FLC incorporates a simple, rule-based IF X AND Y THEN Z approach for solving control problem rather than attempting to model a system mathematically. The FLC model is empirically based, relying on an operator's experience rather than their technical understanding of the system. For example, rather than dealing with temperature control in terms such as "TEMP =500 ^oF", "TEMP < 1000 ^oF ", or "210 ^oF <TEMP< 220 ^oF", terms like "IF (process is too cool) AND (process is getting colder) THEN (add heat to the process)". FLC is capable of mimicking this type of behavior but at very high rate.

2.4 How Does Fuzzy Logic controller Work?

Fuzzy Logic controls requires some numerical parameters in order to operate such as what is considered significant error and significant rate-of-change-of-error, but exact values of these numbers are usually not critical unless very responsive performance is required.

2.5 Here are some guide lines by which a Fuzzy logic controller is designed

1. Define the control objectives and criteria: What is the control objectives? What are the parameters of the system to be controlled? What kind of response is needed? What are the possible (probable) system failure modes?

2. Determine the input and output relationship and choose a minimum number of variables for input to the FLC engine (typically error and rate of change of error for fuzzy PD Controller).

3. Using the rule-based structure of FLC, break the control problem down into a series of IF X AND Y THEN Z rules that define the desired system output response for a given system input conditions. Although it is possible to use a single, instantaneous error parameter without knowing its rate of change, this cripples the system's ability to minimize overshoot for a step inputs.

4. Create FLC membership functions that define the meaning (values) of Input/Output terms used in the rules.

5. Test the system, evaluate the results, *tune the rules and membership functions*, and continuously simulated until satisfactory results are obtained.

2.6 Architecture of Fuzzy Logic Controller (FLC)

A general architecture of fuzzy logic controller is shown in figure 2.1. Various subsystems of FLC are described in the following sections.

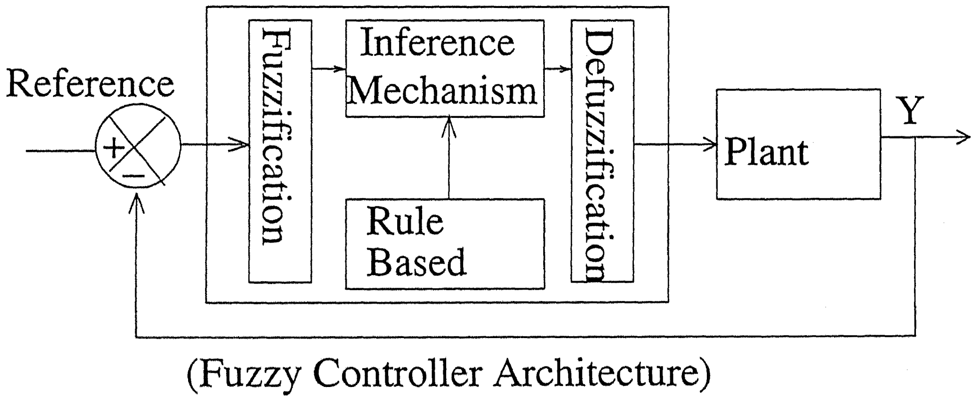


Figure 2.1: Fuzzy Architecture

2.6.1 Fuzzification

As shown in the figure 2.1, fuzzification is a process in which by the crisp inputs are mapped into corresponding linguistic (fuzzy) variables. These linguistic variables are used for making decisions. Each linguistic variable or Fuzzy set has a certain membership function. The membership function assigns a certain degree of membership or weight for each of the linguistic variables. The membership value is nothing but the truth of belongingness of a certain input to that fuzzy set or linguistic variable.

2.6.2 Designing the rule base

IF-THEN Rules

Fuzzy sets and fuzzy operators are the subjects and verbs of fuzzy logic. These IF-THEN rule statements are used to formulate the conditional statements that comprise fuzzy logic.

A single fuzzy IF-THEN rule assumes the form if x is "A" then y is "B". Where "A" and "B" are linguistic values defined by fuzzy sets on the range (universes of discourse) x and y respectively. The if-part of the rule "x is A" is called the antecedent or premise, while the then part of the rule y is "B" is called the consequent or conclusion.

A double fuzzy IF-THEN rule base is if x is "A", y is "B" than z is "C". A sample 81 fuzzy rule base is shown in the Table 2.1. First row of table indicate fuzzy sets for first variable (A). First column of the table indicate fuzzy sets for second variable (B). Other elements of the table indicate rule for the generating control variable.

Where nvl, nl, nm, ns ze, ps, pm, pl, pvl are the fuzzy sets for A named as negative very large, negative large, negative medium, negative small, zero, positive small, positive medium, positive large, positive very large respectively. dnvl, dnl, dnm, dns dze, dps, dpm, dpl, dpvl are the fuzzy sets for B named as negative very large, negative large, negative medium, negative small, zero error, positive small, positive medium, positive large, positive very large respectively. unvl, unl etc are the rules for generating control input. Each of these variable are assigned with a specified value. All these variables are assigned by a specific value.

rules	nvl	nl	nm	ns	ze	ps	pm	pl	pvl
dnvl	unvl	unvl	unvl	unl	unl	unm	unm	uns	uze
dnl	unvl	unvl	unl	unl	unm	unm	uns	uze	ups
dnm	unvl	unl	unl	unm	unm	uns	uze	ups	upm
dns	unl	unl	unm	unm	uns	uze	ups	upm	upm
dze	unl	unm	unm	uns	uze	ups	upm	upm	upl
dps	unm	unm	uns	uze	ups	upm	upm	upl	upl
dpm	unm	uns	uze	ups	upm	upm	upl	upl	upvl
dpl	uns	uze	ups	upm	upm	upl	upl	upvl	upvl
dpvl	uze	ups	upm	upm	upl	upl	upvl	upvl	upvl

Table 2.1: FAM Table for Rule Base

2.6.3 Fuzzy Inference Mechanism

Fuzzy inference is the process of formulating the mapping from a given input to an output using fuzzy logic. The mapping then provides basis from which decisions can be made, or patterns discerned. Now control decisions can be made based on the linguistic variables. It performs two functions. Firstly assigns degree of membership or weight to the variables and secondly determines the output decisions or output linguistic variables based on input linguistic variables.

2.6.4 Defuzzification Model

This converts the fuzzy set of output variables to crisp numerical outputs. These crisp values are the final control inputs to the system to be controlled.

2.7 FLC Design

The design process of an FLC may be divided into the following steps described below.

2.7.1 Selection of Control Variables

The selection of control variables (controller inputs and outputs) depends on the nature of the controlled system and the desired outputs. It is more common to use the output error (e) and the rate or derivative of the output ($\dot{e}$) as controller inputs.

2.7.2 Membership Function

Each of the FLC input signals, output signals and fuzzy variables, have the real line "R" as the universe of discourse. In practice, the universe of discourse is restricted to a comparatively small interval [Xminj, Xmaxj]. The number of fuzzy sets for each fuzzy variable varies according to the application. A common and reasonable number of variables are odd numbers. Increasing the number of fuzzy sets results in a corresponding increase in the number of rules. A membership function is assigned to each fuzzy set [9]. The membership function maps the crisp set into fuzzy set. A set of membership functions defined for nine linguistic variables NVL, Nl, NM, NS, Z, PS, PM, PL, PVL which stand for negative very large, negative large, negative medium, negative small, zero, positive small, positive medium, positive large, positive very large respectively.

Membership function can be of a variety of shapes, the most popular structures are defined as triangular, trapezoidal, or a bell shaped. For simplicity, it is assumed that the membership functions are symmetrical and each one overlaps with the adjacent functions by 50%. In practice, the membership functions are normalized in the interval [-L, L], which is symmetrical around zero. Thus control signal amplitudes (fuzzy variables) are expressed in terms of controller parameters (gain). A triangular membership function is given in the figure 2.2

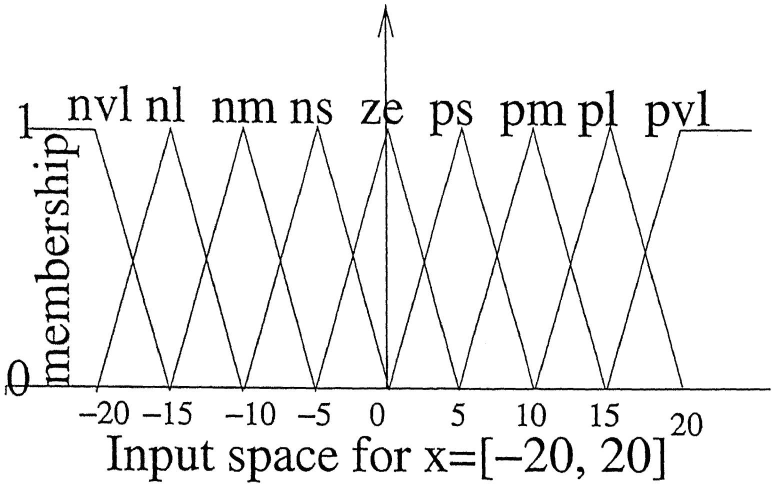


Figure 2.2: Triangular Membership Function for 9 Regions

2.7.3 Defuzzification Strategy

Defuzzification is a process of converting the FLC inferred control actions from fuzzy values to crisp values. This process depends on the output fuzzy set, which is generated from the fired rules . The output fuzzy correlation is formed by either minimum encoding or the product encoding.

Commonly defuzzification methods have been discussed below.

Centroid Method

This procedure also known as center of area or center of gravity method is the most prevalent and physically appealing of all the defuzzification methods. It is given by the algebraic expression

$$\tau = \frac{\sum_{i=1}^{n} \mu_i \omega d\omega}{\sum_{i=1}^{n} \mu_i d\omega}$$

Weighted average method

This method is only valid for symmetrical output membership functions. It is formed by weighting each membership function by respective membership value. This is written in mathematical expression as.

$$\tau = \frac{\sum_{i=1}^{n} \mu_i \omega}{\sum_{i=1}^{n} \mu_i}$$

where

n=number of rules.

μ_i=membership for i_{th} rule.

ω=value of that particular rule.

τ= Control variable.

Max membership principle

This method also known as height method. This scheme is limited to peaked output function. This method is given by the algebraic expression

$\mu_i(\tau) \geq \mu_i(\omega)$

2.8 Summary

FLC offers several unique features that make it good choice for many control problems.

- FLC was conceived as a better method for sorting and handling data. It has proved to be a excellent choice for many control system applications since it mimics human control logic. It can be built into anything from small, hand made products to large computerized process control systems.

- Fuzzy based control (FLC) has become highly competitive due to its better performance, high reliability, robustness, low power consumption and cheapness.

- The thinking process involved in fuzzy realm is not complex. It is simple, elegant and easy to apply.

- In this context, FLC is one of the methodologies for solving control system problem. It lends itself for implementation in systems ranging from simple, small, embedded micro controllers to large, networked, multi-channel PC or workstation-based data acquisition and control systems. It can be implemented in hardware, software, or a combination of both.

- FLC provides a simple way to arrive at a definite conclusion based upon vague, ambiguous, imprecise, noisy, or missing input informations. FLC approach to control problems mimics how a person would make decisions at a much faster rate.

- It is inherently robust since it does not require precise, noise-free inputs. The output control is a smooth control function despite a wide range of input variations.

- Since the FLC processes user defined rules governing the target control system, it can be modified and tweaked easily to improve or alter the system performance drastically. New sensors can easily be incorporated into the system simply by generating appropriate governing rules.

- FLC is not limited to a few inputs or few outputs. This allows the sensors to be inexpensive and imprecise. It keeps the overall system cost effective and low

complexity.

- Because of the rule-based operation, any reasonable number of inputs can be processed and numerous outputs can be generated. Although defining the rule becomes complex if too many inputs and outputs are chosen for a single implementation. It would be better to break the control system into smaller chunks and use several smaller FLC that can be distribute on the system.

- FLC can be used to control nonlinear systems that would be difficult or impossible to model mathematically. This opens doors for control systems designers that would normally be deemed infeasible for automation.

- FLC provides a different approach to control complex systems. This method focuses on what the system should do rather than trying to model how it works. One can concentrate on solving the problem rather than trying to model the system mathematically, if that is even possible.

- On the other hand the fuzzy approach requires a sufficient expert knowledge for the formulation of the rule base, the combination of the sets and the defuzzification.

2.9 Genetic Algorithm (GA)

It is adaptive heuristic search algorithm premised on the evolutionary ideas of natural selection and genetic. The basic concept of GA is to simulate processes in natural system necessary for evolution, specifically those that follow the principles first laid down by Charles Darwin of "survival of the fittest". As such they represent an intelligent

exploitation of a random search within a defined search space to solve a problem.

Genetic algorithm is a search and optimization method developed by mimicking the evolutionary principles and chromosomal processing in genetics. It is a part of evolutionary computing which is a rapidly growing area of artificial intelligence. GA uses a chromosomal representation which required the solution to be coded in a finite length string.

It follows the following steps for minimizing the cost function by optimizing some parameters.

1. Representation.
2. Reproduction.
3. Crossover.
4. Mutation.

The flow chart of **genetic algorithm** is shown in figure 2.3.

2.10 Genetic Algorithm for Single Objective Optimization

Genetic algorithm identifies the parameters and its limit which are to be optimized for a given cost function (performance index). An example for a single objective optimization is illustrated below.

Within the range of selection around 200 children are generated randomly by taking help of binary conversion. Out of these around 20 better children are selected for

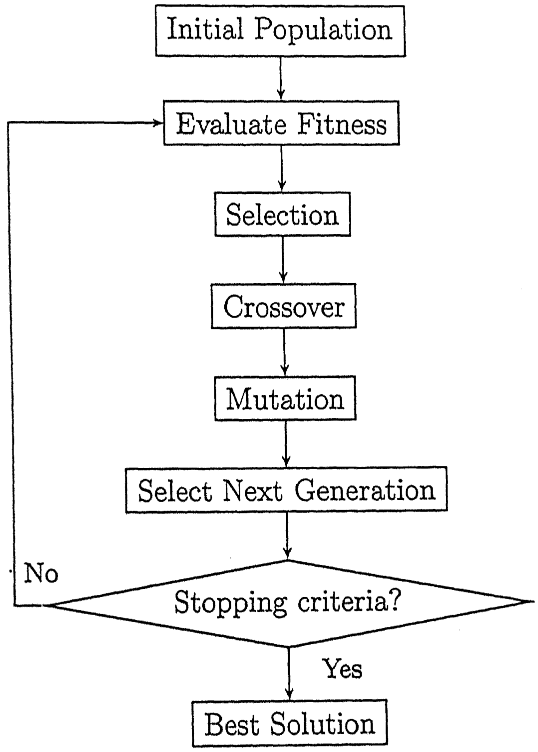


Figure 2.3: Basic Flow Chart of Ganetic Algorithm

minimizing objective function. By permutation and combination again 200 different population are generated for next generation by the process of crossover. Mutation is performed with less density than crossover for finding out new search. The number of generation is found till all the children have same chromosome (same value). Then it is found that no further development of the performance index is possible. The parameters at this performance index are chosen to be optimized value.

2.10.1 Advantages

It uses the natural selection process. For every generation, it takes better values of the parameter for next generation. So it is completely focus based search. It uses crossover and mutation for searching best performance.

2.10.2 Limitation

It can search up to variation of 3 parameters. Some times it shows local minima when limit of the parameters are large. This can not be used for more than one objective function where most of the problems are multi objective function.

2.11 Nsga2.code:

It is used for solving a multi objective optimization. It is a code available in the web site of "Dr. K. Deb, IIT Kanpur". In my Problem it has been tried to set the parameter such that it minimize the error and the control input throughout the simulation.

Chapter 3

Level Controller for a U Tube Steam Generator

3.1 Introduction

Poor control of steam generator water level of a nuclear power plant may lead to frequent nuclear reactor shut downs. This shutdown is more common at low power operation. There is a need to investigate this problem so that a suitable controller for water level regulation can be designed. This work is concerned to study and design of a suitable controller for a U Tube Steam Generator (UTSG) of a Pressurized Water Reactor (PWR). It has a time varying dynamics.

One of the objectives for a steam generator in a nuclear power plant is to control the water level at a desired value by regulating the feed water flow rate. The conventional feed water control scheme cannot provide satisfactory performance within the required wide operating range of 0 to 100% of the specified load. A large proportion of the reactor shutdown causes a severe economic loss. It has been mainly caused by ineffective feed water control. Therefore a better control scheme is required. Past experiences on PWR plant operation, had indicate that frequent reactor trips are due to poor control of water level in steam generator. During low power operation, the level control is complicated

by the thermal reverse effects known as "shrink and swell". Due to the destabilizing vapor content in the tube bundle region, the water level measured in the down comer temporarily reacts in a reverse manner to water inventory change. Increased feed water flow adds mass to the SG, which would be expected to increase the measured down comer water level. It does not increase at high power. But at low power, the cold feed water addition can cause a decrease in the vapor content of tube bundle. A shift in the liquid from the down comer to tube bundle causes a temporary decrease in water level (shrink). Similarly, a decrease in feed-water flow can cause a temporary increase in water level (swell). These reverse effects are confusing for either manual or automatic operation. The only true indication of water inventory change is the flow mismatch between steam and feed-water.

The water level of the steam generator must not be allowed to rise too high, in order to prevent the excessive moisture carryover and the pressure buildup of the containment due to discontinuity of secondary side flow loop. Also, the low water level should be prevented in order to avoid the uncovery of the U-tubes in the secondary side. Therefore, the control of the steam generator water level is important to determine power plant dynamics due to change in operating load.

For the given system FLC and LQR controller are designed and simulated. There are some reports on automatic operation of the steam generator water level from low power to high power range [2].

3.2 Model Development

A steam generator model is a complicated nonlinear dynamical system. The controller design and its performance in actual plant strongly depend on the accuracy of the mathematical model. A highly accurate model of a steam generator is complex and nonlinear. So it is desired that the model should be simple and at the same time relatively accurate for describing the process of dynamics of U Tube Steam Generator (UTSG).

The difficulty in designing the effective water level control for the steam generator arises from a number of factors which are given below.

(a) The inverse response of the plant at low operating power due to the "swell and shrink".

(b) Variation of the plant dynamics with the operating power.

(c) Unreliable flow measurements at low power which preclude effective use of feed-forward control.

We have taken a model which is widely used by many researcher for the control purpose. This is a fourth order model whose parameters are dependent on reactor power level.

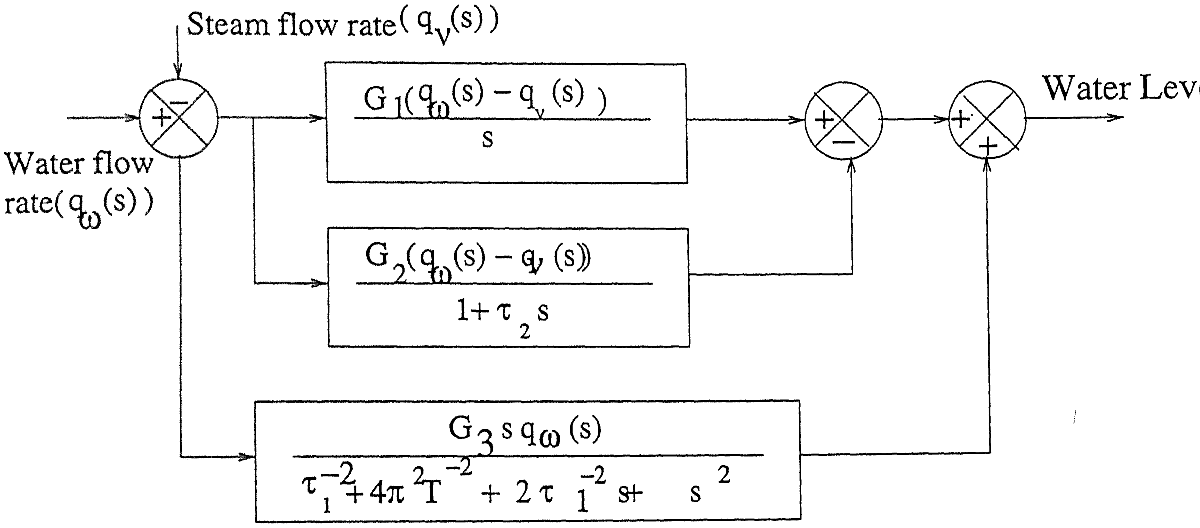


Figure 3.1: Block diagram of boiler level control

3.3 Dynamic Model of PWR for Level Measurement

Transfer function of the water level control system of a UTSG is shown in the figure 3.1. It can be written in mathematical form as,

$$Y(s) = \frac{G_1(q_w(s) - q_v(s))}{s} - \frac{G_2(q_w(s) - q_v(s))}{1 + \tau_2 s} + \frac{G_3 s q_w(s)}{\tau_1^{-2} + 4\pi^2 T^{-2} + 2\tau_1^{-2} s + s^2} \tag{3.1}$$

where,

Y(s) : Water level in meter.

q_w : Feed water flow rate in kg/sec.

q_v : Steam flow rate in kg/sec.

τ_1, τ_2 : Damping time constants in second.

T : Oscillating period in second.

G_1/s is the mass capacity effect of the UTSG. It integrates the flow error $q_w(s)-q_v(s)$ to calculate the change in water level. This term accounts for the level change due to feed water inlet to steam generator and the steam outlet from it. This quantity G_1/s is the actual water capacity which critically affects the removal capability of the primary heat. G_1 is a positive constant and does not depend on load. $G_2/(1+\tau_2 s)$ is the thermal negative effect caused by "swell and shrink". Since these phenomena exhibit exponential responses for step changes of the feed water flow rate and the steam flow rate corresponding dynamics are described by a first order equation. G_2 is a positive constant and is dependent on load. As load increases G_2 decreases. The third term in equation (3.1) is the mechanical oscillation effect caused by the inflow of the feed-water to the U tube steam generator. This is a mechanical oscillation term due to momentum of the water in the down comer. All the water removed from the steam is returned to the down comer and is recirculated. The recirculating water has large momentum acting against relatively small change in flow rate. When the feed water flow rate is suddenly decreased, the water level in the down comer falls initially and then begins to oscillate. This is due to the momentum of the water in the down comer keeping the recirculating flow going down initially and then slowing down. The mechanical oscillation disappears completely after a small multiple of the damping time constant. The variable G_3 is a positive constant and depends upon the load.

The steam generator dynamics has been divided into four linearized regions with respect to operating power level. It is assumed that the dynamics vary linearly over these regions. These variations of the plant parameters with respect to power level are presented in the table (3.1). The actual plant parameters may vary differently, so we study the performance of the controller under the situations when the parameter drifts

q_v(kg/second)	57.4	180.8	381.7	660	1435
P(%power in Mw)	5	15	30	50	100
G_1	0.058	0.058	0.058	0.058	0.058
G_2	9.63	4.46	1.83	1.05	0.47
G_3	0.181	0.226	0.31	0.215	0.105
τ_1	41.9	26.3	43.4	34.8	28.6
τ_2	48.4	21.5	4.5	3.6	3.4
T	119.6	60.5	17.1	14.2	11.7

Table 3.1: Dynamic parameters with respect to operating power

from what is projected by linear interpolation.

3.4 Controller Design for Water Level Control

Systematic approach for linear quadratic regulator (LQR) and fuzzy logic methods have been used to derive the control law where some objectives functions are minimized to derive an optimal controller. The main objective of the controller is to maintain the water level in the steam generator under various operating levels. We also show the effect of parameter drift on the water level through computer simulation results. To design the proposed controller the plant dynamics has been transfered into a suitable state space form. .

3.4.1 State Space Representation of dynamic model

The dynamic equation (3.1) of pressurized water reactor for water level measurement can be splited into four state space equations as,

$$
\begin{aligned}
\dot{x}_1(t) &= G_1(q_w(t) - q_v(t)) \\
\dot{x}_2(t) &= -\tau_2^{-1}x_2(t) - (G_2/\tau_2)(\dot{q_w}(t) - q_v(t)) \\
\dot{x}_3(t) &= -2\tau_1^{-1}x_3(t) + x_4(t) + G_3q_w(t) \\
\dot{x}_4(t) &= -(\tau_1^{-2} + 4\pi^2T^{-2})x_3(t)
\end{aligned}
$$

Output water level is given by

$$y_p = x_1(t) + x_2(t) + x_3(t) \tag{3.2}$$

The state space equation can be written in matrix form,

$$
\begin{aligned}
\dot{x}_p(t) &= A_px_p(t) + B_pq_w(t) + F_pq_v(t) \\
y_p(t) &= C_px_p(t)
\end{aligned} \tag{3.3}
$$

Where,

$$
A_p = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\tau_2^{-1} & 0 & 0 \\ 0 & 0 & -2\tau_1^{-1} & 1 \\ 0 & 0 & -\tau_1^{-2} + 4\pi^2T^{-2} & 0 \end{bmatrix}
$$

$$B_p = \begin{bmatrix} G_1 & -G_2\tau_2^{-1} & G_3 & 0 \end{bmatrix}^T$$

$$F_p = \begin{bmatrix} -G_1 & G_2\tau_2^{-1} & 0 & 0 \end{bmatrix}^T$$

$$C_p = \begin{bmatrix} 1 & 1 & 1 & 0 \end{bmatrix}$$

$$x_p(t) = \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) & x_t(t) \end{bmatrix}^T$$

The matrixes B_p and F_p can be written as

$B_p = [b_1, b_2, b_3, 0]^T$

$F_p = [d1, d2, 0, 0]^T$

3.5 LQR Design for Steam Generator

3.5.1 Introduction

The linear quadratic regulator (LQR) is an optimal control problem where the state equation of the plant is linear, the cost function of the plant is quadratic and the test conditions consist of initial condition of the state and no disturbance inputs. The resulting optimal control known as the linear quadratic regulator. It can be applied to a wide range of control problems due to the flexibility available when selecting the quadratic cost functions [10]. The mathematical derivation of the LQR controller is given in appendix.

3.5.2 Problem Formulation

For achieving asymptotically stable tracking with minimizing quadratic performance index by using LQR [11], the performance index is chosen as,

$$\text{Minimize} J = \int [x_p^T(t) Q x_p(t) + u_p^T(t) R u_p(t)] dt \tag{3.4}$$

subject to the state space equations as given in (3.3),

$$\begin{aligned} \dot{x}_p(t) &= A_p x_p(t) + B_p q_w(t) + F_p q_v(t) \\ y_p(t) &= C_p x_p(t) \end{aligned}$$

It is required that

$$y_p \rightarrow y_r \text{ as } t \rightarrow \infty \tag{3.5}$$

Here, y_r=Water level reference. Its value depend on the design data of U tube steam generator.

Tracking problem is converted into regulator problem by introducing a new state $\dot{q}$

$$\dot{q}(t) = y_p - y_r = C_p x_p - y_r \tag{3.6}$$

After introducing a new state in equations 3.3, the new state space equations in matrix form are given by

$$\begin{aligned} \dot{x}_{pq} &= A_{pq} x_{pq}(t) + B_{pq} u_{pq}(t) + F_{pq} w(t) + H_{pq} y_r \\ y_{pq}(t) &= C_{pq} x_{pq}(t) \end{aligned} \tag{3.7}$$

where,

control input $u_{pq}(t) = q_w$,

$$x_{pq} = \begin{bmatrix} x_p \\ q \end{bmatrix}, \quad A_{pq} = \begin{bmatrix} A_p & 0 \\ C_p & 0 \end{bmatrix}, \quad B_{pq} = \begin{bmatrix} B_p \\ 0 \end{bmatrix}, \quad F_{pq} = \begin{bmatrix} F_p \\ 0 \end{bmatrix},$$

$$C_{pq} = \begin{bmatrix} C_p & 0 \end{bmatrix}, \quad H_{pq} = [0\,0\,0\,0\ -1]^T.$$

The cost function defined in equation (3.4) is thus reduced using equation (3.7), to the following form:

$$J_1 = \int [x_{pq}^T(t) Q_1 x_{pq}(t) + u_{pq}^T(t) R_1 u_{pq}(t)] dt$$

where Q_1 is state weight matrix and R_1 is control weight matrix. To make J_1 non negative, R_1 should be positive definite and Q_1 should be positive definite or at least positive semidefinite. R_1 and Q_1 are selected by the designer. For this particular plant, R_1 and Q_1 are chosen to be unit matrices.

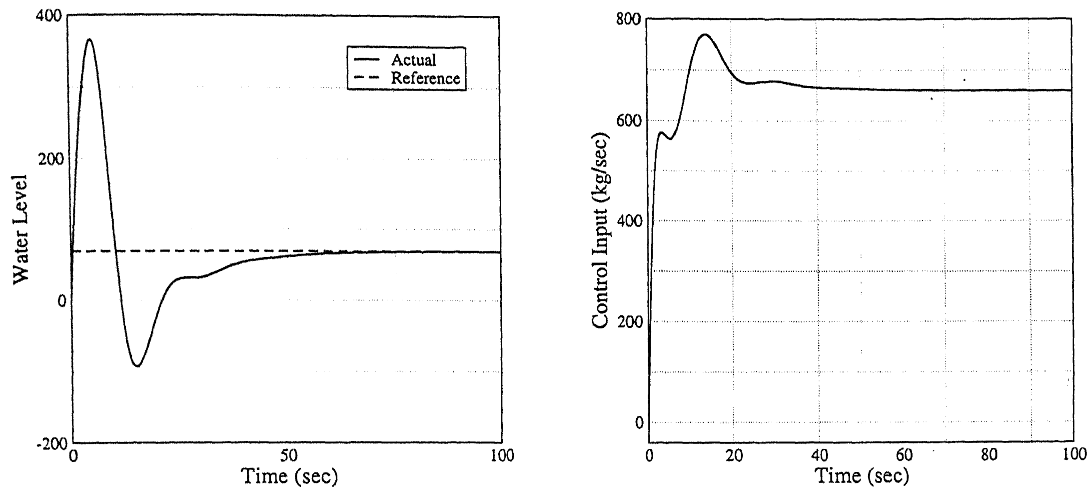
The optimal control input is given by $U_{pq}(t) = -KX_{pq}(t)$

where K is a constant feedback gain matrix given by

$$K = R_1^{-1} B_{pq}^T P_1 \tag{3.8}$$

where P_1 is a constant symmetric positive definite matrix which is the solution of the Algebraic Riccati Equation (ARE) . The (ARE) is given below.

$$P_1 A_{pq} + Apq^T P_1 + Q_1 + P_1 B_{pq} R_1^{-1} B_{pq}^T P_1 = 0 \tag{3.9}$$



(a) water level (m)

(b) water flow rate (kg/sec)

Figure 3.2: LQ tracking at 50% Power

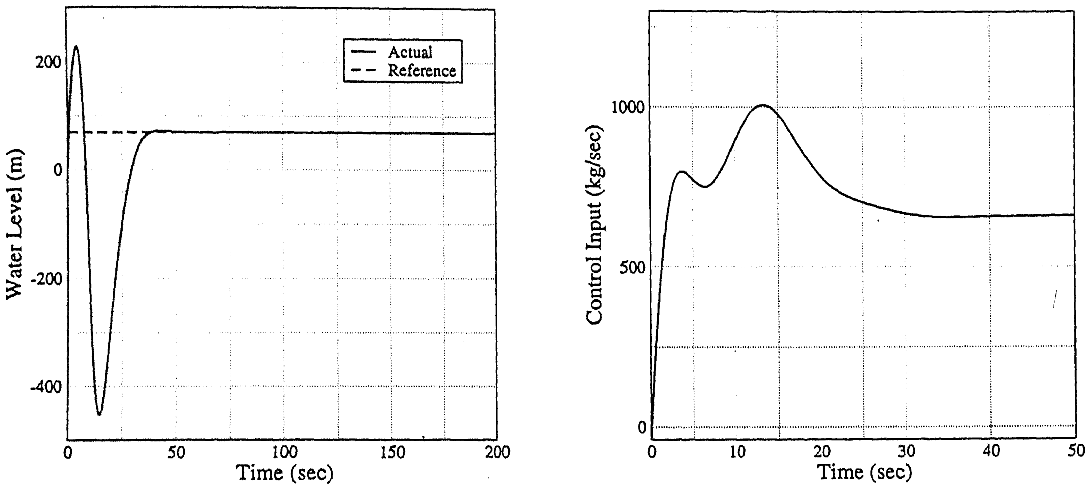
The existence and uniqueness of solution for the above equation are guaranteed by the following assumptions:

1. (A_{pq}, B_{pq}) is a controllable pair.

2. $A_{pq}, Q_1^{1/2}$ is an observable pair.

3.5.3 Simulation Results

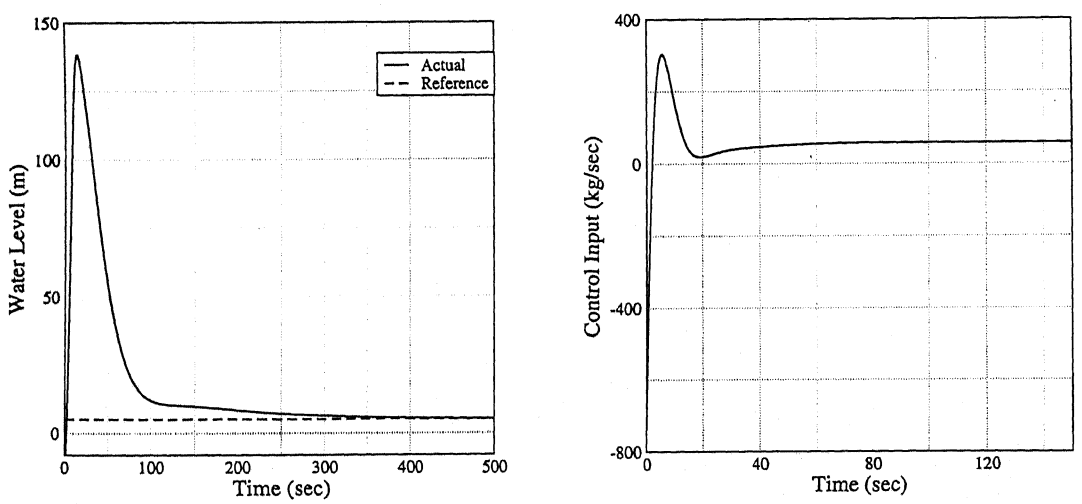
A linear parameter varying model of UTSG of which parameters depends on the reactor power level. The model is linearized over four regions. Referring to table 3.1, the regions are divided according to operating power as: Region I for 0% $\leq$ power $\leq$ 15%, Region II for 15% $\leq$ power $\leq$ 30%, Region III for 30% $\leq$ power $\leq$ 50%, and Region IV for 50% $\leq$ power $\leq$ 100%. Over each region the elements of model matrices are assumed to vary linearly. The simulation studies are carried out for instances when the actual plant parameters drift from the linearized model parameters. It can be seen that variation of



(a) Output (m)

(b) Input (kg/sec)

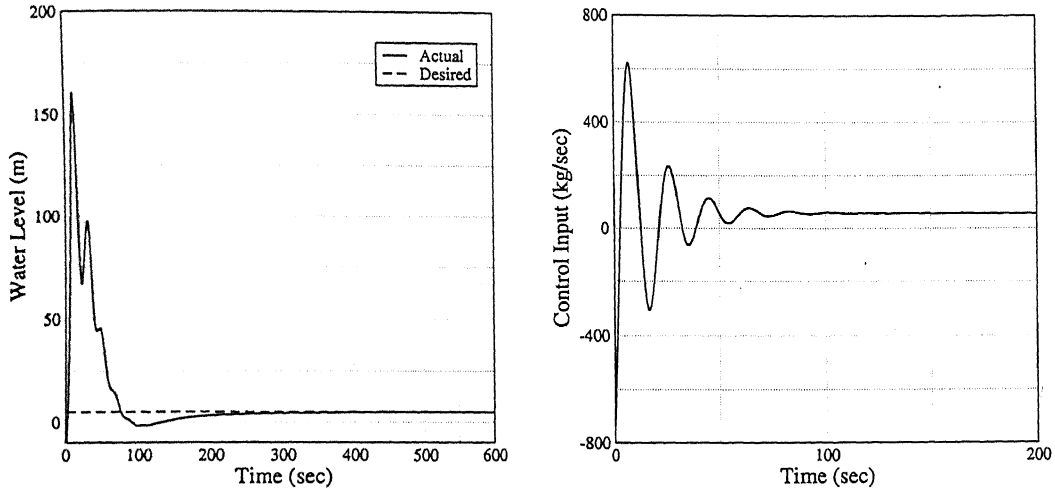
Figure 3.3: LQR Controller at 50% Power at 30% Perturbation



(a) Output (m)

(b) Input (kg/sec)

Figure 3.4: LQR Tracking at 5% Power



(a) Output (m)

(b) Input (kg/sec)

Figure 3.5: LQR Controller at 5% Power at 3% Perturbation

element b_2 of B_p matrix has the most destabilizing effect on the system response. This is because it contains the term G_2 and τ_2 whose value determine the non-minimum phase characteristic of the steam generator. The simulation results are displayed in order to show the effects of these two parameters particularly on the effectiveness of the controller and the quality of the system response. As the power of operation becomes higher the effect of perturbation of parameters becomes less. It is assumed that water flow rate and steam flow rate are equal before the change in power demand occurs, that means the change in water level is zero. The simulation result are taken for the the power level of 5% and 50%.

The simulation results are taken for the system response at the same power of operation (5%) but the parameters G_2 and τ_2 are perturbed such that the element b_2 is increased by 3% for 5% power level and 30% for 50% power level. At steady state the water flow is matched with the steam flow which is desired. As shown from figures 3.2, 3.3, 3.4 and 3.5 a substantial change in system behavior is noticed due to the drift

of system parameter, such as larger overshoot, more settling time and more oscillations before settling down to the desired value. Even when the parameter drifts, the controller is capable of maintaining the change in water level to zero and hence the water flow rate change equals the steam flow rate change. If the element b_2 drifts further the controller may not be robust enough to maintain the water level to the desired value. So we can say that the controller is robust enough to maintain the desired system response if the parameter perturbation lies within certain bounds.

For comparison, we have displayed the simulation results. At 50% power the controller can tolerate maximum 30% perturbation. At 5% Power it can tolerate up to only 3% perturbation.

3.6 FLC with GA Optimization

3.6.1 Introduction

In conventional control, the amount of control is determined in relation to a number of inputs using a set of equations to express the entire control process. Expressing human experience in the form of a mathematical formula is a very difficult task. Fuzzy logic provides a simple tool to interpret this experience into reality. In recent years, fuzzy logic has emerged as a powerful tool and is starting to be used in various applications. The application of fuzzy logic control technique appears to be most suitable one whenever a well-defined control objective cannot be specified and the system to be controlled is a complex one or its exact mathematical model is not available. Fuzzy Logic Controllers (FLCs) are robust and have relatively low computational requirements. They could be constructed easily using a simple microcomputer.

In present work a fuzzy logic controller optimized with genetic algorithm has been designed for controlling the water level of U tube steam generator. Input parameter for the fuzzification plant are

$$e(t) = y_r - y_p(t) \tag{3.10}$$

Now taking the derivative of the equation 3.10 we get

$$\dot{e}(t) = 0 - y_p(t) \tag{3.11}$$

where $e(t)$ is the error in water level which is a function of time,

Y_r is reference water level in meter, which is a constant value. The value of y_r depends on load.

$y_p(t)$ is actual water level in meters.

3.6.2 Fuzzification

According to the fuzzy control strategy crisp inputs $e(t)$ and $\dot{e}(t)$ are converted to fuzzy inputs. For fuzzification, triangular membership functions are chosen with 9 regions. There are two inputs to the fuzzification plant. Total 81 rules are generated [12]. Maximum numbers of rules are fired for a given value of e, $\dot{e}$ are 4.

3.6.3 Inference mechanism

For generating a rule, two fuzzy sets are associated. Individual rule based inference mechanism has been used. In this mechanism the idea is to compute the overall value of the output variable based on the individual contributions of each rule in the rule

base. Each such individual contribution represents the value of the output variable as computed by the single rule only. In this problem, minimum of ($\mu(e)(t)$, $\mu(\dot{e})(t)$ has been chosen as the truth value of a n^{th} rule to be fired.

3.6.4 Defuzzification Module

It is described in section 2.7.3. In this problem, weighted average method [13] has been used for defuzzification. According to this, the control action is computed as,

$$q_w = \frac{\sum_{i=1}^{n} \mu_i \omega}{\sum_{i=1}^{n} \mu_i} \tag{3.12}$$

where,

q_w=water flow rate (control input).

n=number of rules.

μ_i=membership for i_{th} rule.

ω=value of that particular rule.

Finally this value has to be renormalized so that the point wise value of the control output is converted to its physical value.

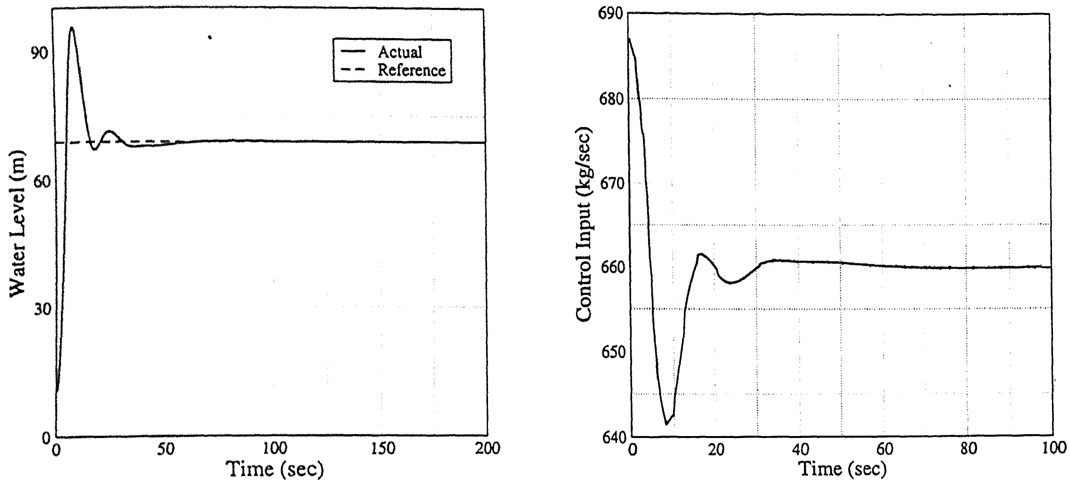
Genetic Algorithm for multi objective optimization (Nsga2 code) has been used for optimizing tool for the FL [14]. The parameters have been optimized are,

1. Membership function for $e(t)$ (error in water level)

2. Membership function for $\dot{e}(t)$ (change in error in water level)

3. Rules for q_w (water flow rate)

The following cost functions are opted to minimize the control input and error,



(a) Output (m)

(b) Input (kg/sec)

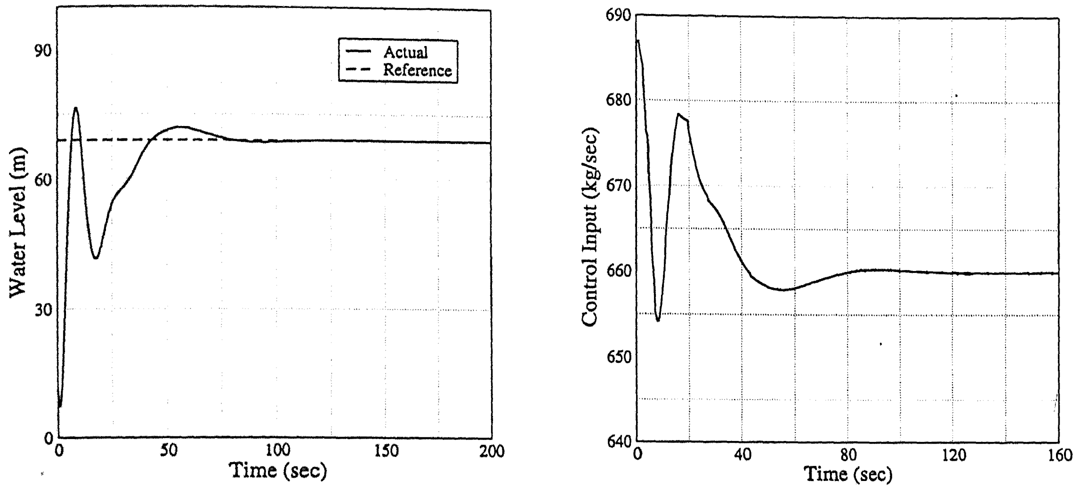
Figure 3.6: Water Level with FL optimized by GA at 50% Power

$$minimize \sum_{i=0}^{t} e^2(t) \tag{3.13}$$

$$minimize \sum_{i=0}^{t} q_w^2(t) \tag{3.14}$$

3.6.5 Simulation Results

As described in section 3.5.3, the values of parameters are chosen accordingly. The FLC has been tested successfully at 50% of power and 50% of power with 3% perturbation of b_2 element of B_p matrix . At steady state the water flow is matched with the steam flow which is desired. As shown from figures 3.6 and 3.7 a substantial change in system



(a) Output (m)

(b) Input (kg/sec)

Figure 3.7: Water Level of U Tube Steam Generator by FL optimized by GA at 50% Power at 3%perturbation

behavior is noticed due to the drift of system parameters, such as more settling time before settling down to the desired value. Even when the parameter drifts, the fuzzy logic controller is capable of maintaining the change in water level to zero. If the element b_2 drifts further the controller may not be robust enough to maintain the water level to the desired value. So we can say that the controller is robust enough to maintain the desired system response if the parameter perturbation lies within certain bounds.

Level of PWR	Settling time in second
Fuzzy controller with 50%power	55
Fuzzy controller with 50%power,3%perturbation	75
LQR controller with 50%power	38
LQR controller with 50%power,30%perturbation	75
LQR controller with 5%power	300
LQR controller with 5%power,3%perturbation	320

Table 3.2: Controller for Fuzzy and LQR Controller

3.7 Summary

Control of UTSG water level strongly affects nuclear power plant availability. To attain a robust and stable control system, two controllers have been presented. LQR and fuzzy logic controllers have been implemented for UTSG water level control of a nuclear reactor. These controllers have been tested from 0% to 100% of the load. Both the controllers perform well when large-scale perturbation occurs. It is clear from table 3.2 that the performance of LQR is better than the fuzzy logic controller in terms of settling time. It is shown that the tracking error can be reduced to zero if the parameter perturbation is bounded within a certain value. Simulation results are provided for some cases to validate the effectiveness of the controller. After implementing the controllers, the water flow matches the steam flow, under various operating conditions and parameter uncertainties. The oscillations of water level and the settling time at 50% power level are less than that at 5% power level, which implies that the control at low power is difficult. At high power operation, the performance of the controller is better. The controller enables the level to be kept within limits and the transient phenomena are overcome within reasonably short time.

Chapter 4

Design of a Steam Generator Level Control Loop for Pressurized Heavy Water Reactor

4.1 Introduction

Nuclear power generation is considered to be one of the major electric power source in future. The schematic diagram of nuclear power station is shown in figure 4.1. These power plants are very sensitive to any disturbances, (i.e, mechanical and electrical) due to various technical and safety reason. U tube steam generator is a major part of nuclear power production. It form a vital link between heat source (the reactor) and the heat sink (the turbo generator system) in most of the nuclear power plants. The schematic diagram of a U tube steam generator for of 500MW is shown in figure 4.2.

Steam generator level control (S.G.L.C.) is designed to provide continuous heat sink to the reactor and ensures the reactor safety during the power production [15]. During reactor shutdown, the system requires removal of the heat. At any power level and under steady state condition, mass balance is achieved as well as level is maintained constant at desired set point. Due to any reason if the level control is lost the mass balance is affected. Level may rise and fall. If water level rises very high, moisture

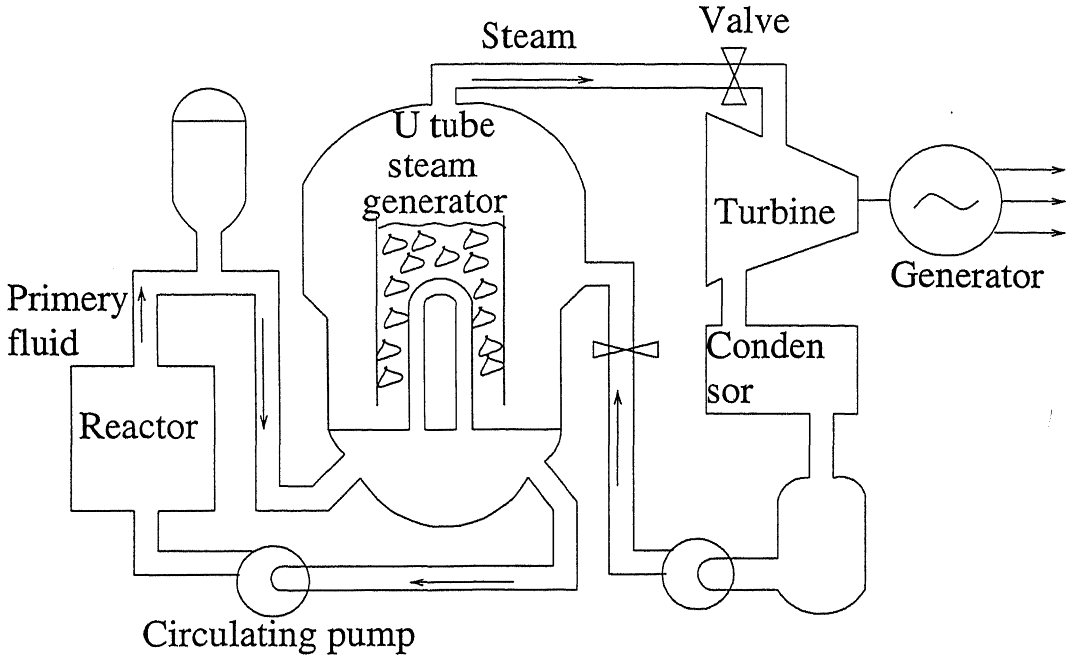


Figure 4.1: U tube steam generator and its environment

carry over into the turbine may occur. Hence turbine trip is proposed at very high level. If water level goes very low, it will affect heat removal and may affect reactor safety due to reduction in heat sink capability. For avoiding such situation, reactor step back is proposed at low level and reactor trip is proposed at very low level in the Steam generator.

In the process control loop, controller provides adjustable parameters which are adjusted to get the required system performance. If all the parameters are not set properly it may lead to poor stability and in the worst case instability. Study of the control loop by varying these parameters in a physical plant is usually not suitable due to various constraints. In simulation the physical system is replaced by its mathematical model. Response of the system towards various control schemes have been studied.

The objective is to develop a simulation package for the steam generator level control loop for 500 MW PHWRs. It will help to study the transient behavior of the loop and

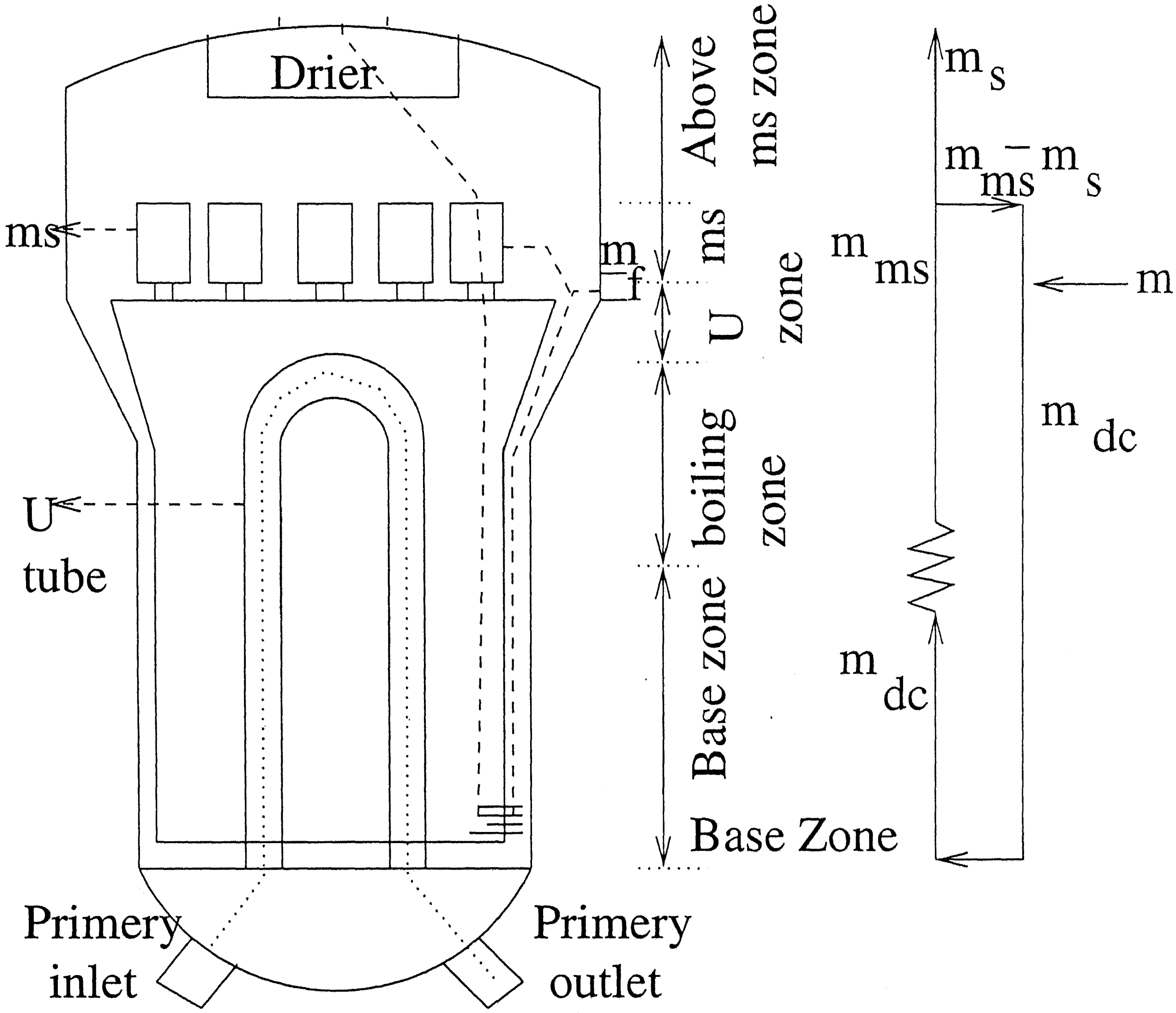


Figure 4.2: Schematic diagram of nuclear steam generator for 500MW power

aid in establishing the parameter values for optimum performance based on various performance indices.

4.2 Description of the System

Nuclear steam generator (SG) is a complex system. As shown in figure 4.2, the nuclear steam generator of 500MW PHWR is U tube type and consists of an integral tube bundle in a shell, moisture separator (MS), dryer and down comer region. Feed water

is pumped into the down comer region of SG through boiler feed pumps. Feed water is mixed with recirculated saturated water (coming from MS) and enter into heat exchange section. Heat rises in the shell around the tube carrying heavy water. Heat transfer takes place and the feed water gets heated up. Steam and water mixture is formed. This water goes to the moisture separator. Steam and water get separated and bulk of water is recirculated through the down comer. Steam along with remaining water (carry over) goes to dryer which further improves the steam quality. This steam is allowed to expand in the turbine or dumped directly to the condenser or vented to the atmosphere [16].

A simplified model of the steam generator water level is simulated. The following assumptions are made for the analysis.

(a) Conservation of mass.

(b) Conservation of energy (enthalpy).

(c) Being a base load plant the heat input from the PHT system is constant.

(d) Feed water temperature is constant.

(e) Pressure in the system is an independent parameter and is uniform throughout the steam generator. Pressure is different at different electrical power. The profile of pressure with change in power is assumed to be known.

The steam generator is divided into three parts:

(a) Non boiling zone with a fixed boiling boundary. Temperature in this zone varies uniformly.

(b) From boiling boundary to average height of U tubes, steam quality varies linearly with height.

(c) Above average height of U tubes to the level in MS (moisture separator), steam quality is assumed to be uniform.

4.3 Factors Affecting the Level Change

Change in level inside steam generator may occur because of change in

(a) Feed water flow/Steam Flow.

(b) Steam Generator Pressure.

4.3.1 Effect of Feed Water Flow on Level Change

At steady state, steam outflow and feed water inflow are same. Due to change in feed water inflow, there will be disparity between two flow rate and hence change in inventory over time. This will result in the level change. Since the enthalpy of feed water is less than that of saturated water. With the change in feed water flow, steam quality also changes. So it will change the homogeneous density and hence there will be change in water level.

4.3.2 Effect of Pressure on Level Change

Saturation temperature is a function of pressure. Hence the part of steam will be converted to water and vice versa, depending on the change in pressure from steady state condition. This will change the steam quality, homogeneous density and hence

level due to void collapse. Density of water, saturated steam and saturated water are function of pressure. So change in pressure will change the level directly.

4.4 Model Development

Based on the assumption of mass and enthalpy balance, a simplified model of steam generator level control is simulated in the following way.

The change in steam quality due to change in feed water flow is calculated. The capacity of the system is accounted. Change in steam quality due to pressure change inside the steam generator is calculated. Overall effect of the above two changes are accumulated to find out the resultant steam quality.

Due to change in steam quality, the volume change in the moisture separator is calculated. Finally the new level for steam generator is calculated considering volume changes.

4.4.1 Different Parameters of this Model

The following parameter values are taken for designing the steam generator of UTSG of 500MW Nuclear power plant.

Internal shell diameter = 2.197m (meters)

Internal Shell diameter in MS region = 3.35m

External diameter of U tubes = 0.019m

Number of U tubes = 2475

U tube height in steam generator =10.86m

Steam generator moisture separator level = 13.09m

Internal diameter of moisture separator = 0.48m

Number of moisture separators = 23

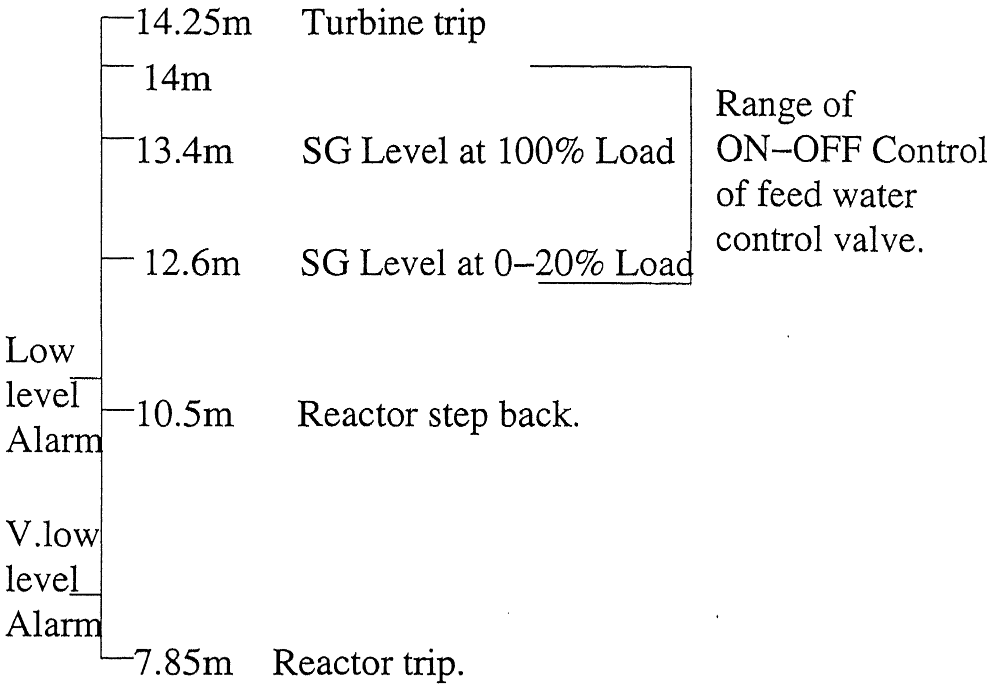


Figure 4.3: Control and Protection set point of Steam Generator Level

4.4.2 Design specification

Maximum pressure of the boiler=75kg/cm^2

Pressure at 0% to 25% power =60kg/cm^2

Pressure varies linearly from 25% to 100% of power

Pressure at 100% power=41.9kg/cm^2

Height of water Level for 0% to 20% of power=12.6m

Height of water Level for 100% of power=13.4m

Height varies linearly from 20% to 100% of power.

The variation of pressure water level of steam generator is shown in figure 4.4.

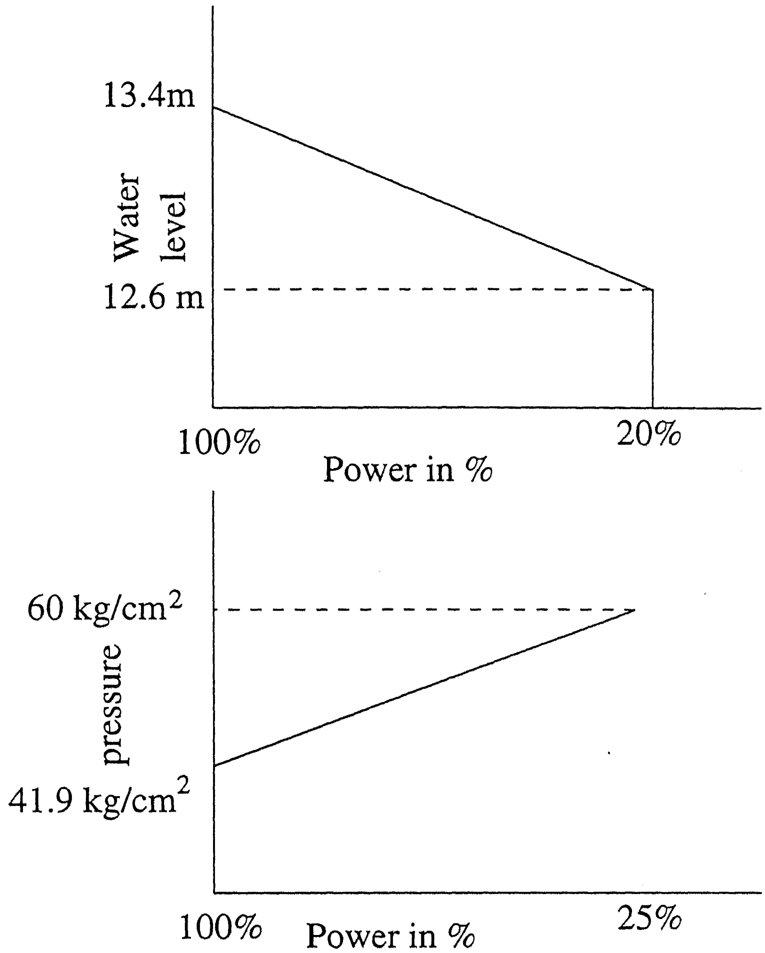


Figure 4.4: Variation of pressure and water level with power

4.4.3 Transfer Function of the Process

The process block is a part of the overall level control block as shown in figure 4.6. Water input is given to the steam generator and it has certain water level to be measured. The process of the steam generation is given in figure 4.5. As shown in this block diagram the whole process is divided into different blocks.

steam flow rate.

The block K_1 is a constant gain which is converting the mismatch between water flow rate and steam flow rate to change in volume.

The block K_2 is a constant gain which converts change in volume to to change in water

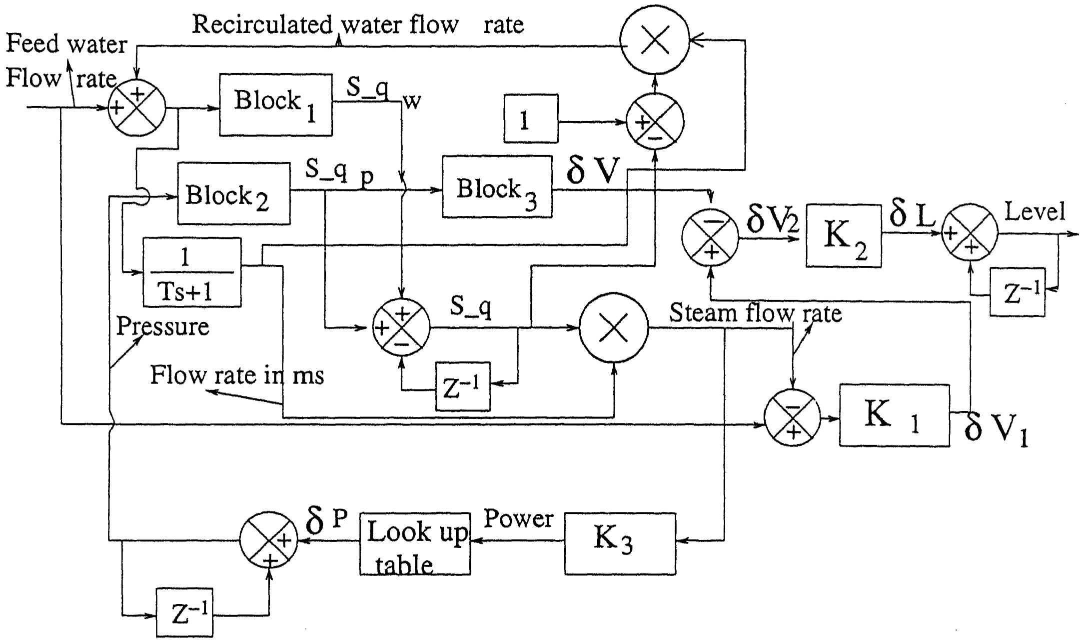


Figure 4.5: Process block diagram of steam generator

level.

The block K_3 is a constant gain which converts steam flow rate to the outlet electrical power.

The different parameters used in this block diagram are, δV δV_1 and δV_2 the changes in volume ($meter^3$).

δP is change in pressure ($\frac{N}{meter^2}$)

S_q , S_{qw}, S_{qp} are the steam qualities.

There are some terms related to process of steam generation as given in **appendix**. In the analysis of this process for the nuclear power plant the following two parameters are chosen to be constant.

Temperature of feed water to the steam generator (T_{Fw})= 180^0C.

The dynamics of $Block_1$, $Block_2$ and $Block_3$ are described as below.

Mathematical Model of Block 1

Inside the stem generator when the unsaturated steam is passed through the moisture separator the water is separated and recirculated . The temperature of the recirculated water is the saturation temperature of water. The recirculated water is mixed with feed water and converted to steam .

Let the different parameters of the steam generator are written as,

F_{ms}= Water and steam flow rate in moisture separator (kg/sec).

S_f=Steam flow rate (kg/sec). $F_w(t)$=Feed water flow rate (kg/sec).

$R_w(t)$=Recirculated water flow rate (kg/sec) .

$W_f(t)$=Net water flow rate (kg/sec).

$T_i(t)$=Temperature of net water flow rate (^0C).

T_F=Temperature of feed water (^0C).

T_R=Temperature of recirculated water flow (^0C). It is saturation temperature of water.

$T_s(t)$ =Saturation temperature of feed water (^0Ce).

$L(t)$=Latent heat of vaporization of water (Joule/kg)

$E_w(t)$=Available energy to convert water to saturated water (Joule/sec).

$E_s(t)$=Available energy to convert water to unsaturated steam.

$E(t)$=Available energy to convert saturated water to unsaturated steam (Joule/sec).

$S_q(t)$=Steam quality.

$S_{qw}(t)$=Steam quality due to change in water flow rate.

S=Specific heat of water. It is taken as as 4.18 kilo Joule/kg in this problem.

The value of the above mentioned parameters are calculated as

$$S_q(t) = \frac{Steam}{Water + Steam}$$

$$S_f(t) = F_{ms} \times S_q(t)$$

$$R_w(t) = F_{ms} \times (1 - S_q(t))$$

$$W_f(t) = R_w(t) + F_w(t)$$

The temperature of water after recirculation is given by,

$$T_i(t) = \frac{F_w(t) \times T_F + R_w(t) \times T_R}{F_w(t) + R_w(t)} \quad (4.1)$$

Heat exchange takes place from primary side to secondary side of U tube steam generator. It is utilized to convert water into steam. In each iteration there is a change in water flow rate. So there is a change in energy of the water. Finally it results a change in steam quality [17].

The energy gained by water for converting water to unsaturated steam is given by,

$$E_s(t-1) = W_f(t-1) \times (S \times (T_s(t-1) - T_i(t-1)) + L(t-1) \times S_q(t-1)) \quad (4.2)$$

The energy gained by water for converting water to saturated water is given by

$$E_w(t) = W_f(t) \times S \times (T_s(t) - T_i(t)) \quad (4.3)$$

The energy available for converting water into steam can be calculated from equations

(4.2) and (4.3) as,

$$E(t) = E_s(t-1) - E_w(t) \tag{4.4}$$

Energy $E(t)$ is not sufficient for converting all the water inside the steam generator into steam. From equation (4.4), steam quality due to water flow rate can be written as

$$S_{qw}(t) = \frac{E}{W_f(t) \times L(t)} \tag{4.5}$$

From equation (4.5) it can be conclude that the steam quality changes due to change in water flow.

Mathematical Model of Block 2

Due to change in pressure, there is a change in energy in the boiler. Change in energy is responsible for change in steam quality. So there is a change in steam quality due to change in pressure.

As described in earlier section, the steam generator is divided into different zones. The mass in the different zones are,

M_{b_U}= Total mass inside the steam generator from boiling section to U tube section (kg).

In this zone both saturated water and saturated steam are present. Steam quality varies linearly from zero to maximum value (S_q). The average value of steam quality is chosen for simulation.

$M_{U_{ms}}$=Total mass from U-tube to moisture separator (kg) from U-tube to moisture separator. In this zone Steam quality is assumed to be uniform.

M_{ms}=Total mass from moisture separator to steam outlet (kg). in this zone steam

quality is assumed to be uniform .

$\Delta E_p(t)$=Extra energy of water due to change in pressure.(Joule/sec)

$E_p(t)$=Energy of the steam (Joule/sec).

$E_{ps}(t)$= Energy available for converting saturated water to unsaturated steam (Joule/sec).

$S_{qp}(t)$=New steam quality due to change in pressure.

The change in energy of water inside the steam generator due to change in pressure is given by,

$$E_p(t-1) = (M_{b_U}(t-1)\times(1-S_q(t-1)/2)+(M_{U_{ms}}(t-1)+M_{ms}(t))\times(1-S_q))\times(S\times T_{sp}(t-1)) \tag{4.6a}$$

$$E_p(t) = (M_{b_U}(t) \times (1 - S_q(t)/2) + (M_{U_{ms}}(t) + M_{ms}(t)) \times (1 - S_q)) \times (S \times T_{sp}(t)) \tag{4.6b}$$

$$\Delta E_p(t) = E_p(t-1) - E_p(t). \tag{4.6c}$$

The energy of the steam can be written as,

$$E_p(t) = (M_{b_U}(t)/2 + M_{U_{ms}}(t) + M_{ms}(t)) \times L(t) \times S_q(t) \tag{4.7}$$

From equations 4.6 and 4.7, the energy for converting saturated water into steam is derived as.

$$E_{ps}(t) = \Delta E_p(t) + E_p(t) \tag{4.8}$$

Now, the steam quality is found to be

$$S_{qp}(t) = \frac{E_{ps}(t)}{(M_{b_U}(t)/2 + M_{U_{ms}}(t) + M_{ms}(t)) \times L(t)} \tag{4.9}$$

Mathematical Model of Block 3

According to the description given in section 4.4.3, there is a change in steam quality due to change in pressure. This results in change the homogeneous density in the boiler as well as change in volume of water.

The different parameters for calculating the process are given below,

$D_{sw}(t)$=Density of saturated water (N/m^2).

$D_{ss}(t)$=Density of saturated steam (N/m^2).

$D_{Uh}(t)$= Homogeneous density from boiling section to U tube section (N/m^2).

$D_{MSh}(t)$= Homogeneous density from U tube section to steam outlet (N/m^2).

ΔV_1=Volume of water reduced from boiling section to U tube section in $meter^3$.

A_U=Area of cross section in U tube section in $meter^2$

$H_U - H_{bb}$= Height of steam generator from boiling zone to U tube zone (meter).

ΔV_2=Volume of water reduced from U tube to moisture separator region in $meter^3$.

A_{a_U}=Area of cross section in moisture separator ($meter^2$).

$H_{MS} - H_U$=Height of steam generator in moisture separator in meter.

ΔV_3=Volume of water reduced from moisture to steam outlet region in $meter^3$.

A_{ms}=Area of cross section above moisture separator to steam outlet in $meter^2$.

lev= Height of steam generator above moisture separator.

ΔV=Total volume reduced due to change in steam quality.

The homogeneous density of steam and water from boiling section to U tube section

due to pressure in previous iteration is given by,

$$D_{Uh}(t) = \frac{1}{S_{qp}(t)/(2.0 \times D_{ss}(t)) + (1 - S_{qp}(t)/2.0)/D_{sw}(t)} \tag{4.10}$$

$$D_{Uh}(t-1) = \frac{1}{(S_{qp}(t-1)/(2.0 \times D_{ss}(t-1)) + (1 - S_{qp}(t-1)/2.0)/D_{sw}(t-1))} \tag{4.11}$$

$$D_{MSh}(t) = \frac{1}{(S_{qp}(t)/D_{ss}(t) + (1 - St_{qp}(t))/D_{sw}(t)} \tag{4.12}$$

$$D_{MSh}(t-1) = \frac{1}{(S_{qp}(t-1)/D_{ss}(t-1) + (1 - S_{qp}(t-1))/D_{sw}(t-1))} \tag{4.13}$$

Decrease in volume due to change in homogeneous density,

$$\Delta V_1 = A_U \times (H_U - H_{bb}) \times \frac{D_{Uh}(t) - D_{Uh}(t-1)}{D_{Uh}(t)} \tag{4.14}$$

$$\Delta V_2 = A_{a_U} \times (H_{MS} - H_U) \times \frac{D_{MSh}(t) - D_{MSh}(t-1)}{D_{MSh}(t)} \tag{4.15}$$

$$\Delta V_3 = A_{ms} \times Lev \times \frac{(D_{MSh}(t) - D_{MSh}(t-1))}{D_{MSh}(t)} \tag{4.16}$$

$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 \tag{4.17}$$

From equation (4.17) the volume change due to change in pressure is calculated.

Block of look of table

This block convert electrical power to to pressure according to design specification as shown in figure 4.4

4.5 Steam Generator Level Control Scheme (SGLCS)

For steam generator (SG) level control, three elements are controlled, i.e., water level, feed water flow rate and steam outflow from the S.G.,

With the increase in load, steam flow will increase . So there will be more steam bubble in SG, which will cause water to swell or raise rather than fall. Thus there will be a transient increase in the water level. Similarly when steam flow reduces or feed water flow increases, bubble tend to collapse and temporarily reduce their formation. This will cause water level to fall rather than rise because of reduced water usage or increased water supply. This will drive the control signal in wrong direction, if control is done with level signal alone. This phenomenon is called "Inverse Response". It will cause delay in control action which makes the controller design more difficult .

Steam generator level is programmed to increase linearly from no load to full load. The Steam generator level program is shown in figure 4.3. The set point for water level in steam generator has been fixed for safety and protection of nuclear power plant. It varies linearly from 20% of load to full load. The controller has to be designed in such a way that the water level in steam generator should follow the set point in order to avoid the reactor shut down or back stepping of the reactor.

The level is measured by the level transmitters as shown in figure 4.6. Its value is compared with fixed level set point. Resulting error is fed to the controller. From the controller output, signal from the feed water flow is subtracted and fed to the control valve through E/A (electrical to analog) converter. Control valve is modulated to control the flow of water under the steady state and hence the water level in SG is controlled to the designed value.

As the transfer function of the process, i.e., steam generator is very much complicated, the formulation requires special attention. As is shown is shown in the figure 4.6 the transfer functions of various other elements in the control loop, i.e, E/A, control valve (CV) converter, flow transmitter, level transmitter etc are very simple. The

coefficient of the transfer function is worked out for the design information according to the manufacture data. The overall block diagram of SGWL is shown in figure 4.6. The various blocks are associated with it are " primary controller, process, secondary controller, E/A (electrical to analog converter), CV (Control valve), FLOW Tx, Level Tx". The transfer function of each block diagram are described below.

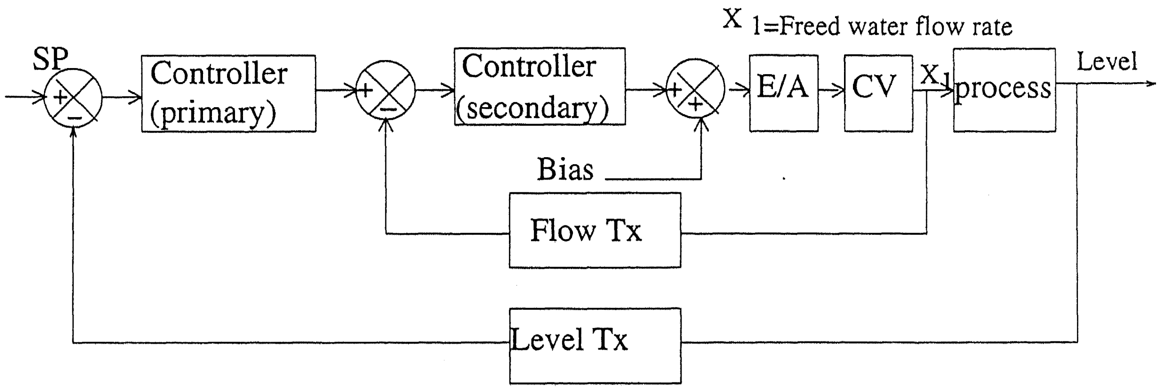


Figure 4.6: Block diagram of steam generator level control

Secondary Controller

As given in the block diagram of steam generator level control in figure 4.6, The secondary controller is chosen as a proportional controller. So its transfer function is,

$$TF = K_{p1} \tag{4.18}$$

Where,

K_{p1}=Gain of the Controller.

E/A Converter (E/A)

Basically electrical to analog (E/A) converter is a delay circuit. There is some time delay in converting the electrical signal to analog signal. So the transfer function is,

$$TF = \frac{G}{1 + sT} \tag{4.19}$$

where,

Gain (G) =1, time constant (T)=0.2 second.

Control Valve (CV)

The control valve is used to open the valve of feed water flow. The transfer function for control valve is

$$TF = \frac{G}{1 + sT} \tag{4.20}$$

where,

Gain (G)=1, time constant (T)=0.4 second.

Feed Water Flow Transmitter (FLOW Tx)

It is basically a transducer which convert feed water flow to the equivalent electrical signal. The output of the flow transmitter is fed to the secondary controller. The

transfer function of the feed water flow transmitter is

$$TF = \frac{G}{1 + sT} \tag{4.21}$$

where,

Gain (G)=1, time constant (T)=0.22 second.

Water Level Transmitter (Level Tx)

It is basically a transducer which converts water level to the equivalent electrical signal. The output of the Tx can be used as as a feed back signal to the primary controller. The transfer function of the feed water flow transmitter is

$$TF = \frac{G}{1 + sT} \tag{4.22}$$

where,

Gain (G)=1, time constant (T)=0.2 second.

4.5.1 Controller Design

As shown in the figure 4.6, a cascade control scheme has been used for water level control of nuclear steam generator. It consists of two controllers. The inner control loop is called secondary control loop and the outer control loop is called primary control loop. The secondary controller is chosen as a proportional controller that is derived for the given problem. The primary controller is chosen as proportional derivative (PD) , proportional integral(PI) and fuzzy logic (FLC) controller.

4.6 Proportional Derivative Controller

A proportional derivative controller has been implemented as a primary controller for the given problem. The error in water level to the primary control loop is given by,

$$e(t) = y_r - y_a(t) \tag{4.23}$$

where ,

y_r=Reference water level in meter,

$y_a(t)$ is the actual water level in meter.

Taking its time derivative, we get

$$\dot{e}(t) = 0 - \dot{y_a}(t) \tag{4.24}$$

The control output from the proportional derivative controller is

$$control = K_p \times e(t) + K_d \times \dot{e}(t) \tag{4.25}$$

where,

K_p= Proportional gain of primary controller.

K_d= Derivative gain of primary controller.

K_{p1}= Proportional gain of secondary controller.

The optimal values of K_p , K_d and K_{p1} are optimized by using **genetic algorithm.** Following cost functions are chosen to minimize the "water flow rate" and "error in

water level".

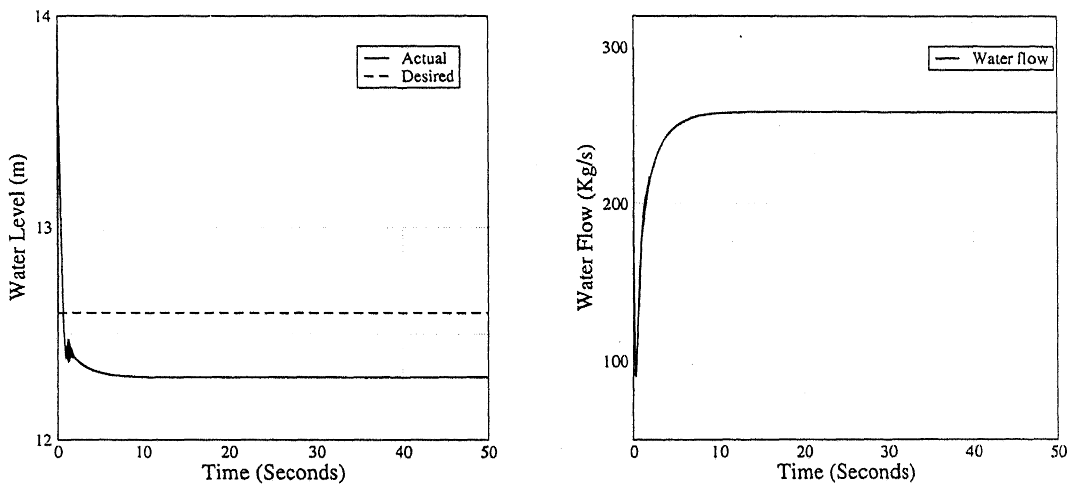
$$minimize \sum_{i=0}^{t} e^2(t) \tag{4.26}$$

$$minimize \sum_{i=0}^{t} q_w^2(t) \tag{4.27}$$

where,

q_w is the water flow rate in kg/sec.

After optimization the value of K_p, K_d and K_{p1} are found to be 599.299622, 80.664223 and 1.191765 respectively.



(a) Water Level

(b) Inlet Water Flow

Figure 4.7: Water Level Controller by Using PD Controller

4.6.1 Simulation Results

The proportional derivative controller optimized with genetic algorithm is implemented for controlling the water level of U tube steam generator. The simulation results are

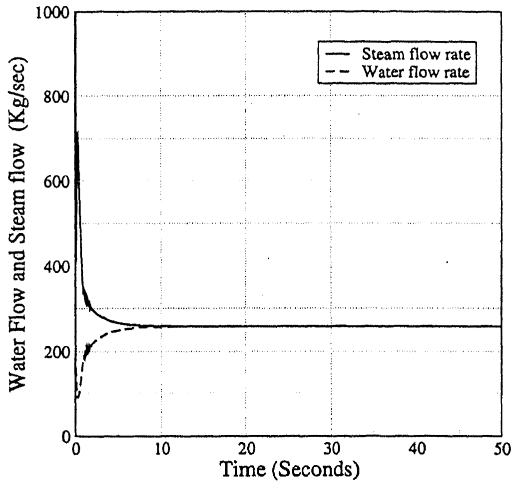


Figure 4.8: Water flow and steam flow rate for PD control

in the figure 4.7. It is observed that, there is no overshoot but some steady state error remains in the desired output. After optimizing the values of parameters for proportional derivative controller gain, there is an improvement of the performance, but the steady state error can not be eliminated completely. Under steady state condition the water flow is matched with the steam flow which is desired. It has been shown in figure 4.8

4.7 Proportional Integral (PI) Controller

A proportional integral (PI) controller has been implemented as a primary controller for the given problem. As described in section 4.6 the error in water level to the primary control loop is given by,

$$e(t) = y_r - y_a(t) \tag{4.28}$$

The control output from the PI controller is,

$$control_1 = K_p \times e(t) + \int_0^t K_i \times \dot{e}(t) \tag{4.29}$$

where,

K_p= Proportional gain.

K_i= Integral gain.

The optimal values of K_p, K_i and K_{p1} are calculated by using **genetic algorithm**. Following cost functions are chosen to minimize the "water flow rate" and "error in water level" during the simulation.

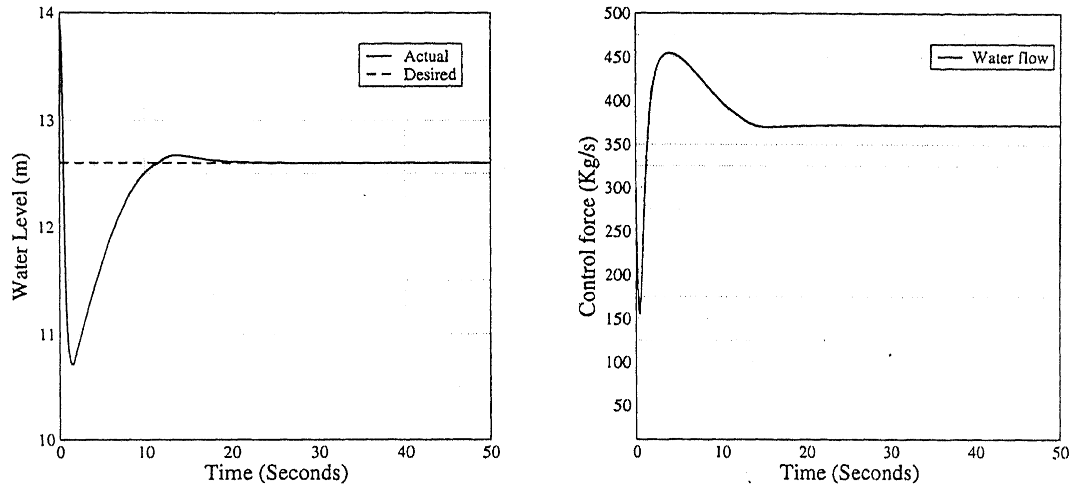
$$minimize \sum_{i=0}^{t} e^2(t) \tag{4.30}$$

$$minimize \sum_{i=0}^{t} q_w^2(t) \tag{4.31}$$

After optimization, the values of K_p , K_i and K_{p1} are chosen to be 249.413925 ,49.970673 and 1.162353 respectively. This output of PI controller is fed as the reference to the secondary control loop.

4.7.1 Simulation Results

The PI controller optimized with genetic algorithm is implemented for controlling the water level of U tube steam generator. The simulation results are shown in the figure 4.9. The set point tracking is achieved by using this controller. It is observed that,



(a) Water Level

(b) Inlet Water Flow

Figure 4.9: Water Level Controller by Using PI Controller

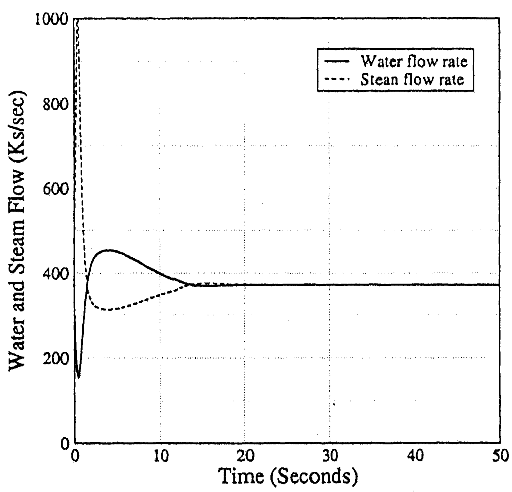


Figure 4.10: Water flow and steam flow rate for PI control

initially there is some overshoot but it damped out after some time. Under steady state condition the water flow is matched with the steam flow which is desired. It is shown in figure 4.10

4.8 Fuzzy Logic Controller

A fuzzy logic controller (FLC) is implemented as a primary controller for the given problem. The mechanism of the fuzzy logic controller is described in chapter 2. As described in section 4.6 the error in water level to the primary control loop is given by,

$$e(t) = y_r - y_a(t) \tag{4.32}$$

As described in earlier section, fuzzy logic controller is applied to this plant. The parameters of fuzzy logic controller such as "membership function for error", "membership function for change in error", "rules" are optimized by using **genetic algorithm**. Following cost functions are opted to minimize the "water flow rate" and "error in water level" through out the simulation.

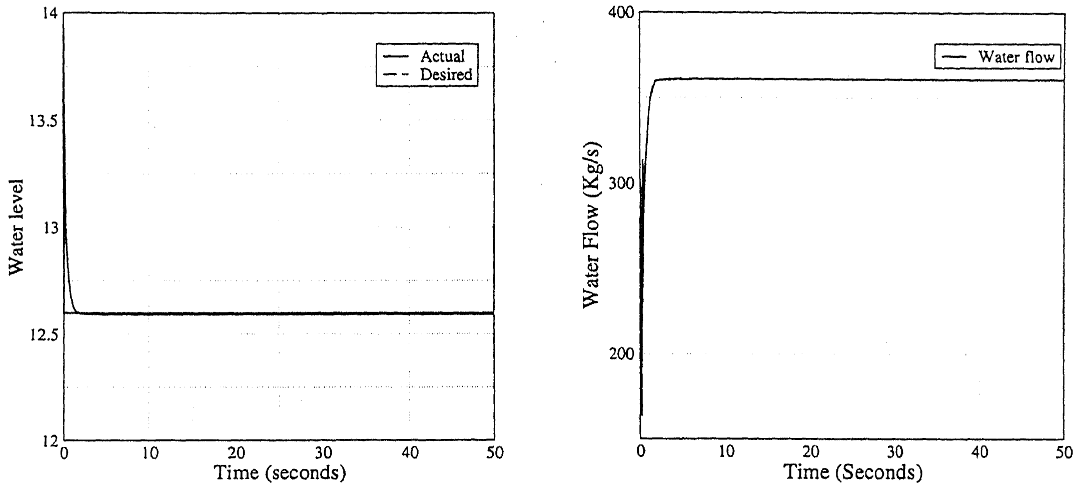
$$minimize \sum_{i=0}^{t} e^2(t) \tag{4.33}$$

$$minimize \sum_{i=0}^{t} q_w^2(t) \tag{4.34}$$

After fuzzification and defuzzification the control output is generated from the fuzzy plant, is fed as a input to the secondary controller.

4.8.1 Simulation Results

The fuzzy controller optimized with genetic algorithm has been implemented successfully for controlling the water level of U tube steam generator. The simulation results are shown in the figure 4.11. It is observed that, there is no overshoot and also steady



(a) Water Level (b) Inlet Water Flow

Figure 4.11: Water Level Controller by Using Fuzzy Controller

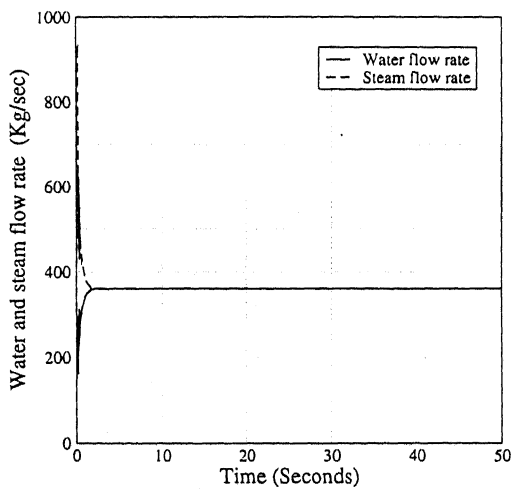


Figure 4.12: Water flow and steam flow rate for FL control

state error is approximately zero . The settling time is least among all the three controllers. Under steady state condition the water flow is matched with the steam flow which is desired. It is shown in figure 4.12

Level of UTSG	Steady state error in %	Overshoot in %	Settling time in sec
Fuzzy controller	0.06	0	1.5
PI controller	0.03	11.11	20
PD controller	2.41	0	2

Table 4.1: Settling Time for Different Controllers

Description of the table

The simulation results of three different controllers have been shown in table 4.1. It shows that for FLC controller, steady state error in % and settling time in second for water level are 0.06 and 1.5 respectively. For PI controller, steady state error in %,overshoot in % and settling time in second for water level are 0.03, 11.11 and 20 respectively. For PD controller, steady state error in % and settling time in second for water level are 2.41 and 2 respectively. It is seen that the performance of fuzzy logic controller is better than other two controllers.

4.9 Validity of the Model

The described scheme is used for developing a nonlinear model of steam generator level control loop for 500 MW PHWR . But the geometry of the steam generator is also different from the one used for 500MW PHWR. Besides this there are some minor differences in designing the steam generator which are ignored in developing the steam generator level model. Parameters are obtained for theoretical observation under optimum performance where as the parameters obtained from practical observations are operating parameters which may not be optimum. There may be some errors in the designed model due to some wrong approximations.

So, the system may try to operate around the theoretical optimum parameters first

by reducing the gain and then by slowly reducing the reset time to further validate the simulation.

4.10 Summary

A nonlinear model of steam generator level control loop for 500 MW PHWR has been proposed. The two controller are used in cascade mode so that full control capacity of the control valve can be utilized. In this scheme the primary controller is placed in the outer loop to control the water level in SG. Output of this controller is set point for secondary controller (Feed-water flow controller). Secondary controller is a proportional controller. The primary controller has been implemented by using fuzzy controller, PI controller and PD controller. By using above controllers, the water level of nuclear steam generator is controlled. The results of all the controllers have been shown in Table 4.1. It is observed that the settling time in case of fuzzy logic controller is smallest than other two controller. In PI controller, steady state error is zero but there is over shoot. In PD controller though there is zero overshoot but some steady state error is remained. But in Fuzzy logic controller there is no over shoot and steady state error is approximates to zero.

Chapter 5

Nonlinear Control System Design: Single Link Manipulator

5.1 Introduction

In recent years considerable amount of interest has been paid by the researchers on the use of fuzzy logic-based controllers. Many successful applications have done and fuzzy logic controller are commercially available. Processes which are it is already commercially available. Processes which are controllable by a skilled operator by using fuzzy logic controller rather than the use of conventional control methods are the target. Single link manipulator is a nonlinear single input single output system. It has one actuator and one degree of freedom. A flexible arm is connected to a motor. The angular motion of the arm depends upon the angular motion of the motor. The input torque is given to the motor which is under our control. So by controlling the torque, the position of the arm can be controlled.

Industrial manipulators are often equipped with conventional PID controllers due to their simplicity in structure and ease of design. However, when using a PID control, it is difficult to achieve a desired level of control performance. One of the most widely used design methods for FLC is to define membership functions of linguistic variables

and to formulate fuzzy rules by control engineers [18]. Design of an efficient fuzzy logic controller involves the optimization of parameters of fuzzy sets and proper choice of the rule base. The parameters of the controller have been optimized based on genetic algorithm.

The objective of the present research is to study more advanced control techniques for a single link manipulator in order to achieve a better performance both in simulation and experimentation. Specifically, a fuzzy logic based control configuration is developed for precise position tracking control of a single link manipulator. various conventional control strategies have been implemented for position tracking.

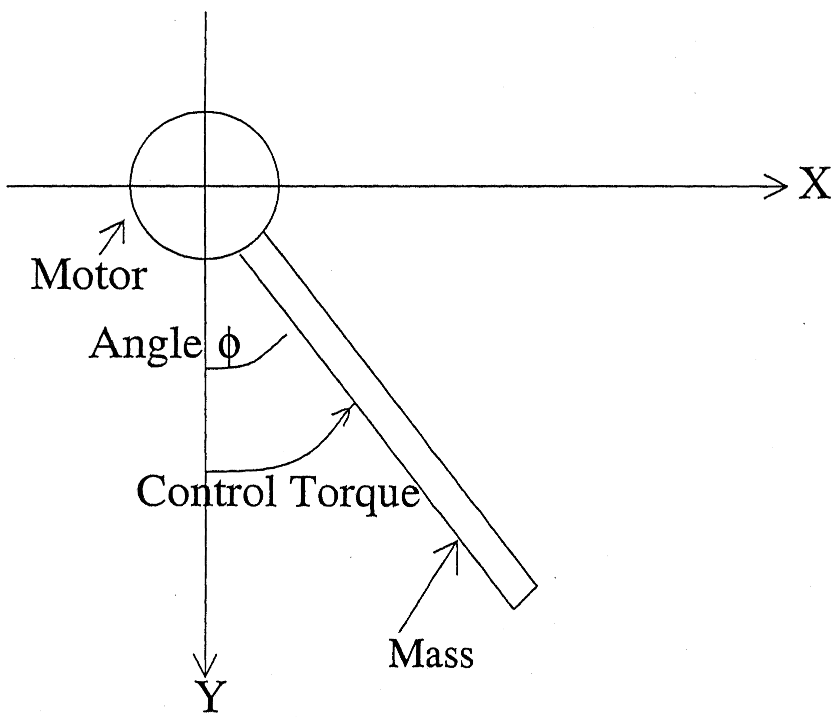


Figure 5.1: Single Link Manipulator

5.2 Model Development

The single link manipulator is shown in figure 5.1. A arm is connected to the motor. The motion of the arm depend upon the motion of the motor. A dynamic model of the manipulator has derived by using Lagrangian formulation of equations of motion. The parameters have been to describe its motion are,

m=Mass of the pendulum in kg.

l=Length of the pendulum in meter.

θ= Angle of the pendulum from vertical bottom position in rad.

g=Acceleration due to gravity in $\frac{meter}{sec^2}$.

τ= Control torque applied to the pendulum in Nm.

5.2.1 Lagrangian formulation of equations of motion

The dynamic equation of single link manipulator has been derived in this section [11]. From the equation of motion, the kinetic energy K of the connecting rod is,

$$K = \frac{1}{2}ml^2\dot{\theta}^2 \tag{5.1}$$

We assume that the potential energy of the pendulum is zero when it is at rest at vertical downward position. Hence the potential energy U of the pendulum at any position is given by the equation,

$$U = mgl(1 - cos\theta) \tag{5.2}$$

The Lagrangian function L is

$$L = K - U \tag{5.3}$$

$$= \frac{1}{2}ml^2\dot{\theta}^2 - mgl(1 - cos\theta) \tag{5.4}$$

Now the Lagrange equation describing the pendulum motion is

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}}) - \frac{\partial L}{\partial \theta} = \tau \tag{5.5}$$

Using the equations (5.4) and (5.5), the dynamics of single link manipulator (SLM) can be written as,

$$ml^2\ddot{\theta} = -mglsin\theta + \tau \tag{5.6}$$

The parameters values are used for performing the simulation are given below

m=1 kg, l=1 meter, $g = 9.81kg/meter^2$.

5.3 Control Objective

The pendulum behaves in to and fro motion which produce sinusoidal oscillation. The control force is generated in such a way that the motion of the pendulum follows sinusoidal trajectory. The block diagram of the control system is shown in Figure 5.2. A reference sine wave is compared with actual position of the single link manipulator (SLM) given as the error in the plant. Different controllers such as proportional derivative, feed back linearization, fuzzy logic controller which have been used are described below.

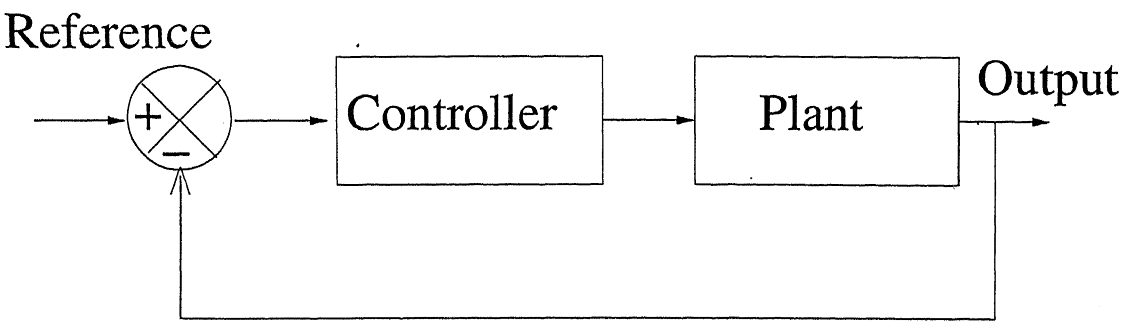


Figure 5.2: Controller for Single Link Manipulator

5.4 Proportional Derivative Controller

A proportional derivative controller has been designed for the given problem. The error signal is,

$$e(t) = sin(t) - \theta(t) \tag{5.7}$$

t is the the time of the response in seconds.

$e(t)$ is the input to the controller. Now taking its time derivative, we get

$$\dot{e}(t) = cos(t) - \dot{\theta}(t) \tag{5.8}$$

The control torque for proportional derivative controller is

$$\tau = K_p \times e(t) + K_d \times \dot{e}(t) \tag{5.9}$$

where,

K_p= Proportional gain.

K_d= Derivative gain.

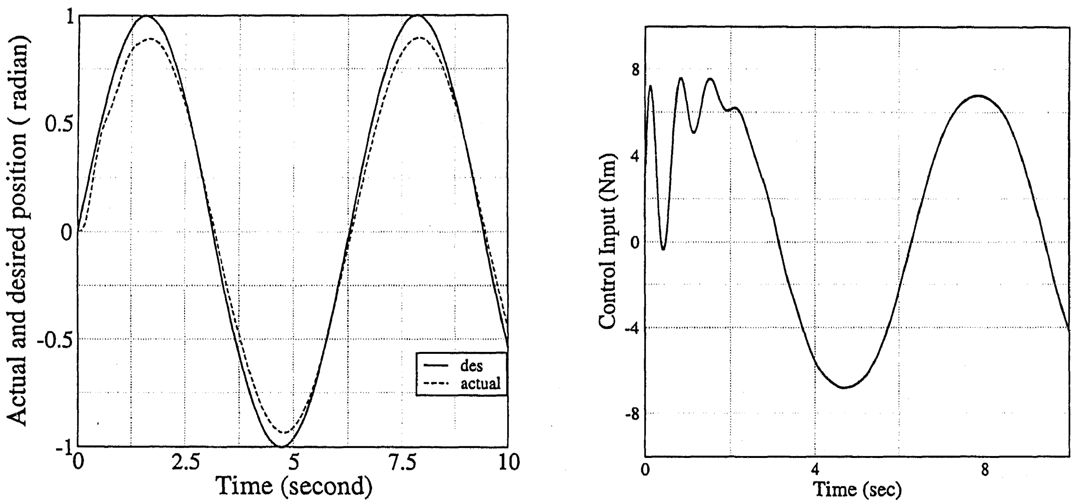
The optimal value of parameters K_p and K_d are calculated by using genetic algorithm. For applying **genetic algorithm** to the given system the following cost functions are

chosen,

$$minimize \sum_{i=0}^{t} e^T(t)k_1e(t) + \dot{e}^T k_2 \dot{e}(t) \tag{5.10}$$

$$minimize \sum_{i=0}^{t} \tau^2(t) \tag{5.11}$$

k_1, k_2 are taken as unity. After optimization, the values of K_p and K_d are found to be 74 and 3 respectively.



(a) Pendulum Angle (rad)

(b) Input (Nm)

Figure 5.3: PD Controller for Single Link Manipulator

5.4.1 Simulation Results

The proportional derivative controller optimized with genetic algorithm has been implemented for designing the controller for single link manipulator. The simulation for this system has been done in C++ language. The simulation results have been shown in the figure 5.3. It is observed that, the desired trajectory of SLM does not follow

perfectly the path of sinusoidal response. There is an steady state error in the output. Initially there is some oscillation but latter it has damped out.

5.5 Feedback Linearization Technique

Feedback linearization is nonlinear control system design procedure which has attracted many researchers in recent years. The central idea of this approach is to algebraically transfer a non linear system dynamics into a (fully or partly) linear one, so that the linear control techniques can be applied. It can be used as model simplifying devices in development of robust or adaptive nonlinear controllers [19]. It has been used successfully to address some practical control problems. These include the control of helicopters, high performance air crafts, industrial robots and biomedical devices.

In this chapter feedback linearization technique is used to designing the controller for single link manipulator. The details of feed back linearization is given in **appendix** The dynamic equation of single link manipulator, given in equation (5.6). It can be written as

$$ml^2\ddot{\theta} + mglsin(\theta) = \tau \tag{5.12}$$

All the symbols are defined in section 5.2. Choosing sin(t) as trajectory, the error can be written as

$$e(t) = sin(t) - \theta(t) \tag{5.13}$$

Taking time derivative of the above equation we get,

$$\dot{e}(t) = cos(t) - \dot{\theta}(t) \tag{5.14}$$

$$\ddot{e}(t) = -sin(t) - \ddot{\theta}(t) \tag{5.15}$$

A function "$r(t)$" is chosen such that

$$r(t) = \dot{e}(t) + \lambda e(t) \tag{5.16}$$

where, λ is a constant multiplying factor.

Taking derivative on both sides of equation (5.16),

$$\dot{r}(t) = \ddot{e}(t) + \lambda\dot{e}(t) \tag{5.17}$$

Putting the value of $\ddot{e}(t)$ from equation (5.14) in equation (5.17) we get

$$\dot{r}(t) = -sin(t) - \ddot{\theta}(t) + \lambda\dot{e}(t) \tag{5.18}$$

Putting the value of $\ddot{\theta}(t)$ from equation (5.12) in equation (5.18) we get

$$\dot{r}(t) = -sin(t) + \lambda\dot{e}(t) - \frac{(\tau - mglsin(\theta))}{ml^2} \tag{5.19}$$

Taking the values of m=1 kg, and l=1 meter, the equation becomes

$$\dot{r}(t) = -sin(t) + \lambda\dot{e}(t) - (\tau - gsin(\theta)) \tag{5.20}$$

τ is proposed in such a way that

$$\dot{r}(t) = -kr(t) \tag{5.21}$$

Where k is a positive constant. The solution of equation (5.22) can be expressed in time domain as

$$r(t) = e^{-kt} \tag{5.22}$$

from equation (5.22) it can be said that,

$$\lim_{t \to \infty} r(t) = 0 \tag{5.23}$$

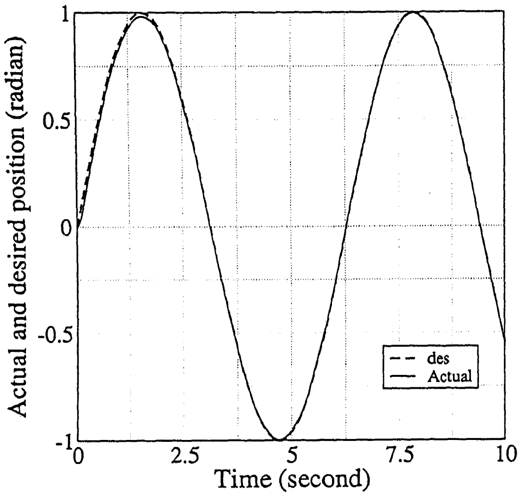
$$\Rightarrow \lim_{t \to \infty} (\dot{e}(t) + \lambda e(t)) = 0 \tag{5.24}$$

From equation (5.24) it can be concluded that

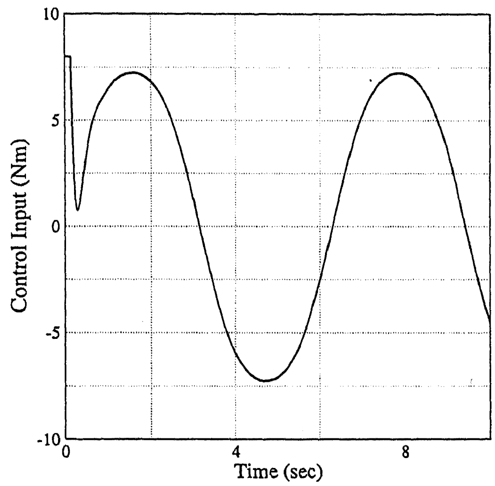
$$\Rightarrow \lim_{t \to \infty} e(t) = 0 \tag{5.25}$$

5.5.1 Simulation Results

The feedback linearization controller has been implemented for designing the controller for single link manipulator. The feed back gain matrix is calculated by using Matlab. The simulation results are shown in the Figure 5.4. It is observed that the trajectory of SLM perfectly follow the path of sinusoidal response. There is no oscillation in the desired trajectory, control action is smooth. The performance of the controller is given



(a) Pendulum Angle (rad)



(b) Input (Nm)

Figure 5.4: Trajectory tracking of SLM by FBL Controller

in table 5.1.

5.6 Fuzzy Controller with GA Optimization

In present work fuzzy logic controller has been designed for sinusoidal tracking of the single link manipulator. Input to the fuzzification plant are taken from error in position and velocity (derivative of position). So it also called as fuzzy PD controller.

5.6.1 Fuzzification

According to the fuzzy control strategy, crisp inputs $e(t)$ and $\dot{e}(t)$ are converted to fuzzy inputs. For fuzzification, triangular membership functions are chosen with 9 region. There are two inputs to the fuzzification module. Total 81 rules are generated [12]. Maximum number of rules are fired for a given value of $e(t)$, $\dot{e}(t)$ are 4.

5.6.2 Inference mechanism

As described in section 3.6.3, individual rule based inference mechanism is used. In this problem, following inference has been chosen.

Minimum of ($\mu(e)(t)$, $\mu(\dot{e})(t)$) is taken as the truth value of n^{th} rule to be fired.

5.6.3 Defuzzification Module

It is described in section 2.7.3. In this problem, weighted average method [13] has been used for defuzzification. According to this, the control action is computed as,

$$\tau = \frac{\sum_{i=1}^{n} \mu_i \omega}{\sum_{i=1}^{n} \mu_i} \tag{5.26}$$

where,

τ=Control input.

n=number of rules.

μ=membership for each rule.

ω=value of that particular rule.

Genetic Algorithm for multi objective optimization (Nsga2 code) has been used as the optimizing tool for FLC [14]. The parameters have been optimized are,

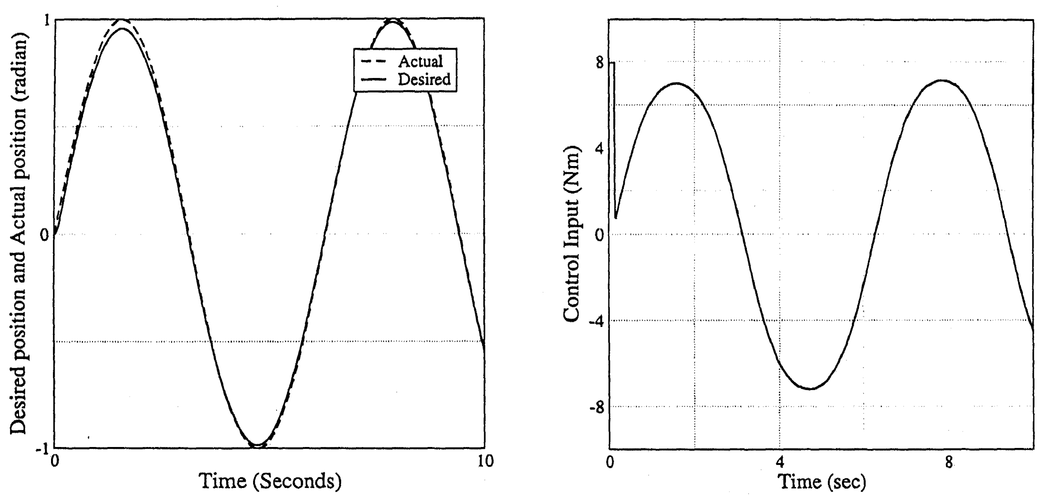
1. Membership function for $e(t)$ (error in position).
2. Membership Function for $\dot{e}(t)$ (error in velocity).
3. Number of rules.

The following cost functions are opted to minimize the torque and error throughout the simulation,

$$minimize \sum_{i=0}^{t} e^T(t)k_1e(t) + \dot{e}^T(t)k_2\dot{e}(t) \tag{5.27}$$

$$minimize \sum_{i=0}^{t} \tau^2(t) \tag{5.28}$$

The values of k_1, k_2 have been taken as 3, and 1 respectively.



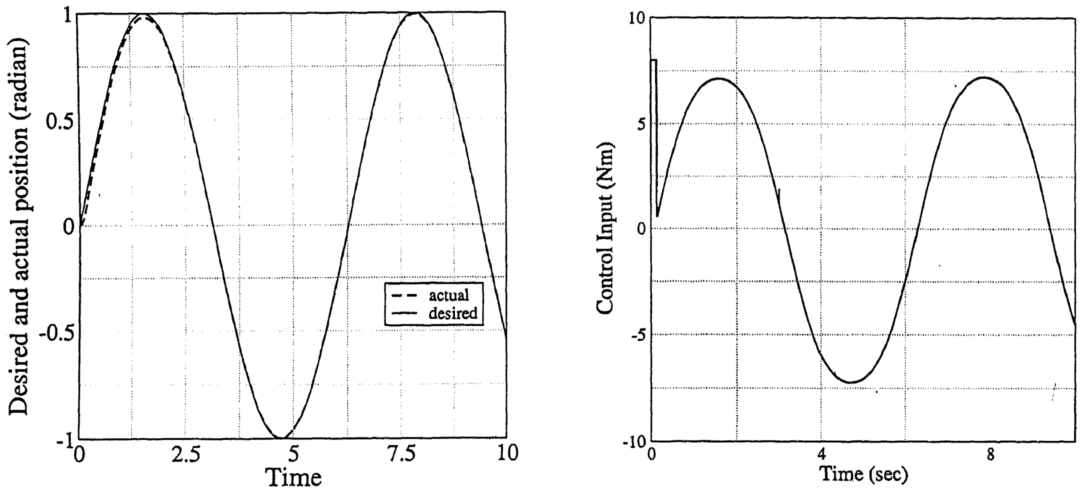
(a) Pendulum Angle (rad)

(b) Input (Nm)

Figure 5.5: FL of Single Link Manipulator

5.6.4 Simulation Results

The simulation results are shown in the figures 5.6 and 5.5. It is observed that the trajectory of SLM follows the path of sinusoidal response. Control action is smooth. Figure 5.5 shows the trajectory without optimization and figure 5.6 shows the trajectory with optimization. From figures 5.5 and 5.6, it cleared that after optimization



(a) Pendulum Angle(rad)

(b) Input (Nm)

Figure 5.6: FL with GA Optimization for Single Link Manipulator

of certain parameters of FLC, the controller gives a better performance. Comparative performance of all the control techniques is given in table 5.1. Table 5.1 shows

Single Link Manipulator	RMS error of position in ^0C	RMS Control force in Nm
Fuzzy controller	0.006818	5.16258
PD controller	0.064365	4.8038
Linearized feedback controller	0.0001125	5.1755

Table 5.1: Trajectory Tracking of SLM

a comparative performance between FLC, PD, and PI Linearized feedback controllers in terms of RMS value of error and control force. It observed from this table that for FLC controller, the root mean square of error of position of pendulum in radian and root mean square of control force in Nm are 0.006818 and 5.16258 respectively. For PD controller, the root mean square of error of position of pendulum in radian and root mean square of control force in Nm are 0.064365 and 4.8038 respectively. For linearized feedback controller, the root mean square of error of position of pendulum in radian and root mean square of control force in Nm are 0.064365 and 4.8038 respectively.

5.7 Summary

The system dynamics of single link manipulator (SLM) as derived using Euler-Lagrange formulation. It has a nonlinear dynamic. Three controllers such as fuzzy logic, proportional derivative, feedback linearized controller have been implemented for getting the sinusoidal trajectory of single link manipulator. Fuzzy logic controller optimized with GA gives a very good trajectory tracking. The control action is less and smooth. This gives us the motivation for using fuzzy controller for more complex non linear systems. Fuzzy logic controller is relatively easy to develop and robust as compared to conventional control strategies.

Chapter 6

Nonlinear Control System Design: Reaction Wheel Pendulum

6.1 Introduction

The Reaction Wheel Pendulum (RWP) is a non linear under actuated system. It is a pendulum with a rotating wheel at the end, which is free to rotate about an axis parallel to the axis of the pendulum. The wheel is actuated by a dc motor, while the pendulum is under actuated [20]. The motor produces a torque on the wheel, causing the wheel to spin. The control torque is generated by the angular acceleration of the disk (pendulum) which is responsible to control the system.

Balancing the pendulum in the vertical upward position is difficult. By using fuzzy logic and state feed back controllers, balancing controllers have been designed in this chapter.

6.2 Model of the Reaction wheel pendulum

m_1 : mass of the pendulum in Kg.

m_2 : mass of the wheel in Kg.

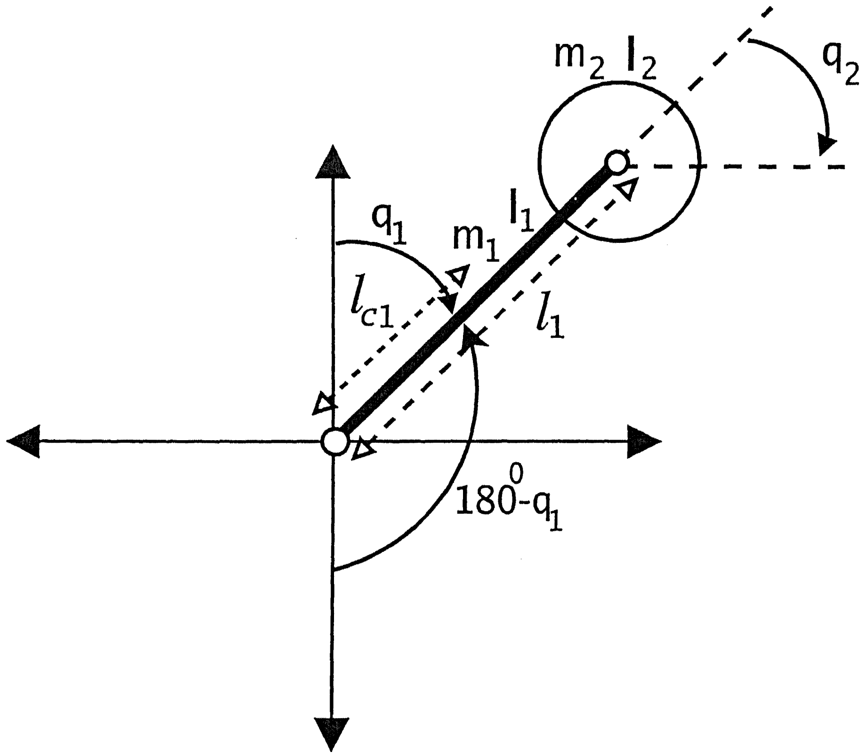


Figure 6.1: Reaction Wheel Pendulum

l_1 : length of the pendulum in m (meter).

l_{c1} : distance from the pivot to the center of the pendulum in m.

I_1 : moment of inertia of the pendulum in Kgm^2 .

I_2 : moment of inertia of the wheel in Kgm^2.

q_1 : pendulum angle with vertical axis in radian.

$\dot{q_1}$=Velocity of pendulum in radian/sec.

q_2 : angle of the wheel in radian.

$\dot{q_2}$=Velocity of the wheel in radian/sec.

τ : motor torque input (control input) applied on the disk in Nm.

6.2.1 Lagrangian formulation of equations of motion

The kinetic energy of the connecting rod alone, is given by

$$K_1 = \frac{1}{2}(m_1 l_{c1}^2 + m_2 l_1^2)\dot{q_1}^2 \tag{6.1}$$

The kinetic energy of the wheel is given by

$$K_2 = \frac{1}{2}m_2 l_1^2 \dot{q_1}^2 + \frac{1}{2}I_2(\dot{q_1} + \dot{q_2})^2 \tag{6.2}$$

From equations (6.1) and (6.2), the kinetic energy of the system is $K = K_1 + K_2$.

where,

$$K = \frac{1}{2}(m_1 l_{c1}^2 + m_2 l_1^2 + I_1 + I_2)\dot{q_1}^2 + I_2 \dot{q_1}\dot{q_2} + \frac{1}{2}I_2 \dot{q_2}^2 \tag{6.3}$$

Total potential energy of the system comes out to be

$$P = (m_1 l_{c1} g + m_2 l_1 g)(cos(q_1) - 1). \tag{6.4}$$

Lagrangian equation of the system is given by $L = K - P$.

$$L = \frac{1}{2}(m_1 l_{c1}^2 + m_2 l_1^2 + I_1 + I_2)\dot{q_1}^2 + I_2 \dot{q_1}\dot{q_2} + \frac{1}{2}I_2 \dot{q_2}^2 - m(cos(q_1) - 1) \tag{6.5}$$

where $m = m_1 l_{c1} + m_2 l_1$

From Euler-Lagrange's equations of motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}(\dot{q}, q)\right) - \frac{\partial L}{\partial q}(\dot{q}, q) = \tau \tag{6.6}$$

where,

q=position in radian,

$\dot{q}$=velocity radian/sec,

τ=control input torque in Nm.

From the equations (6.5) and (6.6), the dynamic equation of reaction wheel pendulum

$$\begin{aligned} (m_1 l_{c1}^2 + m_2 l_1^2 + I_1 + I_2)\ddot{q}_1 + I_2\ddot{q}_2 - mgsin(q_1) &= 0 \qquad (6.7) \\ I_2\ddot{q}_1 + I_2\ddot{q}_2 &= \tau \qquad (6.8) \end{aligned}$$

In compact form, the dynamic equation of the system can be written as

$$D(q)\ddot{q} + g(q) = u \tag{6.9}$$

where,

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}, \quad u = \begin{bmatrix} 0 \\ \tau \end{bmatrix}$$

$D(q)$ is the inertia matrix and is given by

$$D(q) = \begin{bmatrix} m_1 l_{c1}^2 + m_2 l_1^2 + I_1 + I_2 & I_2 \\ I_2 & I_2 \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

$$g(q) = \begin{bmatrix} -mgsin(q_1) \\ 0 \end{bmatrix}$$

Hence the dynamic equation can be written as

$$d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 - mgsin(q_1) = 0 \quad (6.10)$$

$$d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 = \tau \quad (6.11)$$

Pendulum parameters:

The kit for doing real time experiment is available in intelligent control lab. The values of various parameter of the kit have been taken for the simulation of the system.

Mass of the pendulum, $m_1 = 0.2164$ Kg

Mass of the wheel, $m_2 = 0.0850$ Kg

Length of the pendulum, $l_1 = 0.1270$ meter

Distance from pivot to the center of mass of the pendulum, $l_{c1} = 0.1173$ m

Moment of inertia of the pendulum, $I_1 = 2.233\times10^{-4}$ Kgm^2

Moment of inertia of the wheel, $I_2 = 2.495\times10^{-5}$ Kgm^2

Ratings of the drive motor:

Maximum speed of the motor, $N_{max} = 825$ rad/sec.

Electrical Time constant, Te =0.5 ms (milli second).

6.3 State Feedback Controller by Pole Placement

The non linear dyanamics (6.10) and (6.11) are linearized around the unstable equlibrium point which is vertical upward position of the pendulum. A state feedback controller has been designed using this linearized model.

The mathmatical derivation of state feedback controller is given in **appendix**. The system dynamic (6.10) and (6.11) can be written in the following form:

$$\ddot{q_1} = \frac{d_{22}mg}{det(D)}sin(q_1) - \frac{d_{12}}{det(D)}\tau \tag{6.12}$$

$$\ddot{q_2} = -\frac{-d_{21}mg}{det(D)}sin(q1) + \frac{d_{11}}{det(D)}\tau \tag{6.13}$$

where $det(D) = d_{11}d_{22} - d_{12}d_{21}$.

The linearized state space equations for the given plant is given as

$$\dot{X} = AX + B\tau \tag{6.14}$$

$$X = \begin{bmatrix} q_1 & \dot{q_1} & q_2 & \dot{q_2} \end{bmatrix}^T$$

,

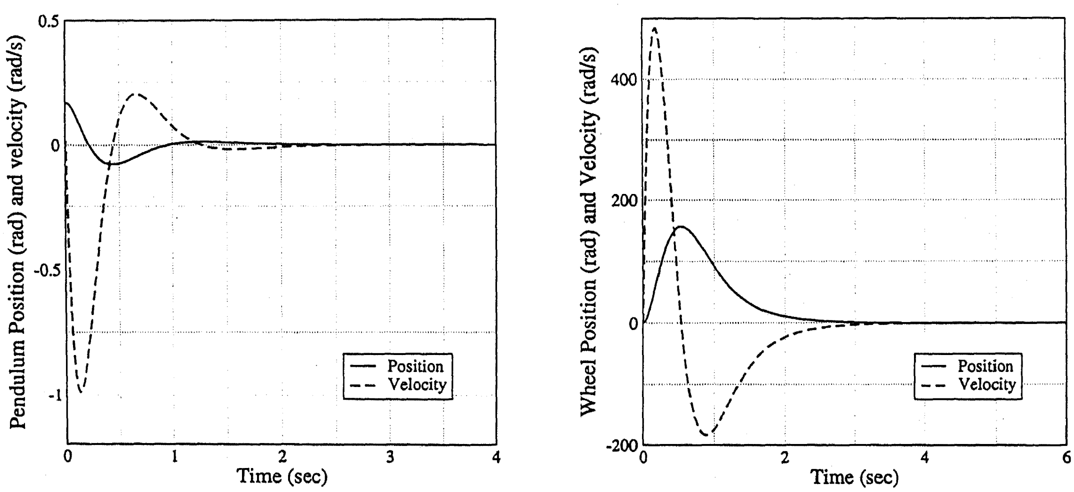
$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & \frac{d_{22}mg}{det(D)} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-d_{21}mg}{det(D)} & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{-d_{12}}{det(D)} \\ 0 \\ \frac{d_{11}}{det(D)} \end{bmatrix}$$

Where X is the state vector, A is the system matrix, B is the conreol matrix and τ is the control torque.

The controllability matrix Q for this system is given by $[B,\ AB,\ A^2B,\ A^3B\ ]$. The rank of the controllable matrix is found to be 4 [21]. Hence the linearized system is controllable. A full state feedback controller has been designed.

The control law $\tau = -KX$ is chosen so as to place the poles of $A - BK$ in the left half of $s - plane$. Then the system becomes asymptotically stable.

where K=$[K_1,\ K_2,\ K_3,\ K_4]$.



(a) pendulum angle (rad)

(b) Wheel Velocity (rad/sec)

Figure 6.2: SFB Controller for Reaction Wheel Pendulum

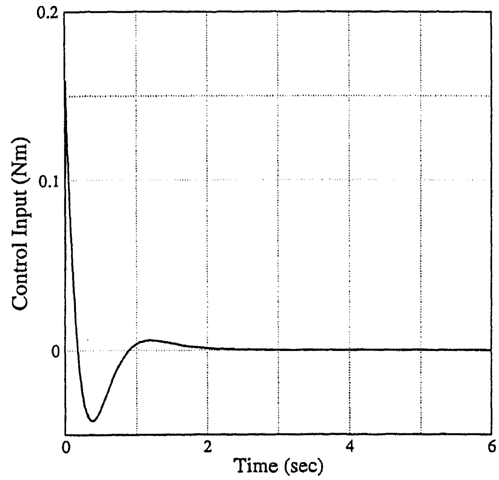


Figure 6.3: Control Force for SFB Controller

6.3.1 Simulation Results

The state feedback (SFB) controller has been implemented for reaction wheel pendulum which is a nonlinear system. The simulation have been shown in the figures 6.2 and 6.3.After settling down the position and velocity of the pendulum and the velocity of the wheel go to zero. It is observed that by using the SFB controller the reaction wheel pendulum is balanced at its inverted top position with a region of attraction of ± 10 degrees. The pendulum is made stable in the vertical upward position and the wheel velocity goes to zero as shown in figure 6.2 The control action is shown in figure 6.3

6.4 Fuzzy Logic Controller for Reaction Wheel Pendulum

A fuzzy logic controller has been designed to stabilize the pendulum at its top unstable equilibrium point. The disk position $q_2(t)$ can be arbitrary, since disk position is of no consequence during the control. For reaching equilibrium point it is required to control

the position of pendulum $q_1(t)$, velocity of pendulum $\dot{q_1}(t)$ and velocity of wheel $\dot{q_2}(t)$. The disk position is left unspecified.

The fuzzy variables are defined as follows

$$e_{q1}(t) = 0 - q_1(t) \tag{6.15a}$$

$$\dot{e}_{q1}(t) = 0 - \dot{q_1}(t) \tag{6.15b}$$

$$\dot{e}_{q2}(t) = 0 - \dot{q_2}(t) \tag{6.15c}$$

where, $e_{q1}(t)$ is the error in position of pendulum, $\dot{e}_{q1}(t)$ is the error in velocity of pendulum and $\dot{e}_{q2}(t)$ is the error in velocity of wheel.

6.4.1 Fuzzification

For fuzzification, triangular membership functions with 9 region are chosen for the fuzzy variables defined in equation (6.15). Total 729 (9^3) rules are generated [12]. Maximum numbers of rules are fired for a given value of $e_{q1}(t)$, $\dot{e}_{q1}(t)$ and $\dot{e}_{q2}(t)$ are 8.

6.4.2 Inference Mechanism

As described in section 3.6.3, individual rule based inference mechanism is used. In this problem, following inference mechanism is chosen.

Minimum of $(\mu(q_1),\ \mu(\dot{q_1}),\ \mu(\dot{q_2}))$ is taken as the truth value of n^{th} rule to be fired.

6.4.3 Defuzzification Module (DM)

It is described in section 2.7.3. In this problem, weighted average method [13] has been used for defuzzification. According to this, the control action is computed as,

$$\tau = \frac{\sum_{i=1}^{n} \mu_i \omega}{\sum_{i=1}^{n} \mu_i} \tag{6.16}$$

Where the terms appeared in equation (6.16) have been described in section 2.7.3. Finally this value has to be denormalized so that the point wise value of the control output is converted to its physical value.

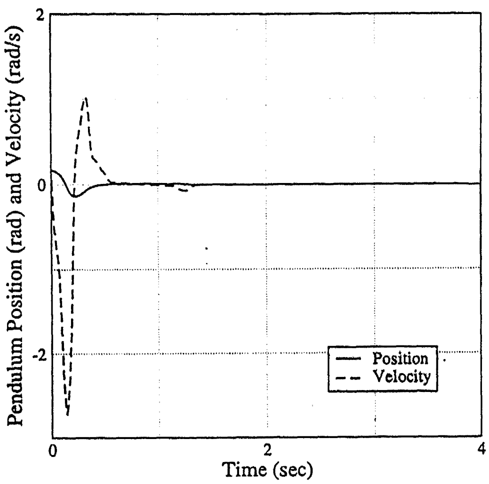
Genetic Algorithm for multi objective optimization (Nsga2 code) is used as a optimizing tool for the fuzzy logic controller [22]. The following parameters of FLC are optimized using GA.

1. Membership function for $e_{q1}(t)$.
2. Membership function for $\dot{e}_{q1}(t)$.
3. Membership function for $\dot{e}_{q2}(t)$.
4. Number of rules.

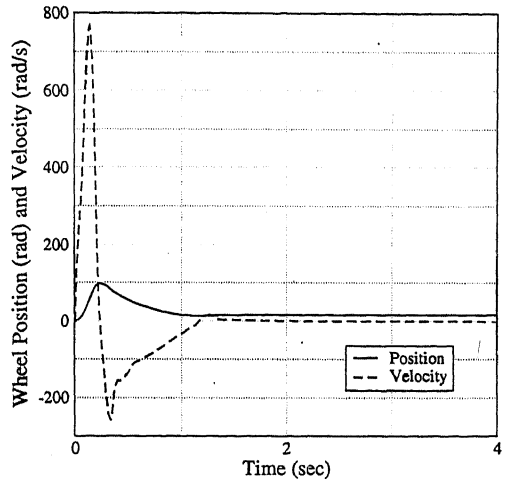
Following cost functions are opted to minimize the torque and error during the simulation.

$$minimize \sum_{i=0}^{t} \dot{e}_{q2}^{2}(t) \tag{6.17}$$

$$minimize \sum_{i=0}^{t} \tau^{2}(t) \tag{6.18}$$



(a) Pendulum Angle (rad)



(b) Wheel Velocity (rad/sec)

Figure 6.4: Trajectory Tracking of Velocity and Position of Pendulum by Using FL Controller

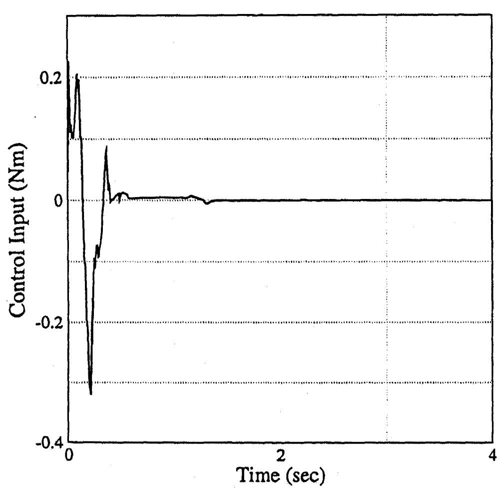


Figure 6.5: Control Torque for FL Controller with GA Optimization

6.4.4 Simulation Results

The fuzzy logic controller (FLC) has been implemented for reaction wheel pendulum which is a nonlinear dynamical system. The simulation results are shown in the figures 6.4 and 6.5. After settling down the position and velocity of the pendulum and the velocity of the wheel go to zero. It is observed that by using the FL controller the reaction wheel pendulum is balanced at its inverted top position with a region of attraction of ±10 degrees. The settling time is less than that of state feed back controller. After settling down the position and velocity of the pendulum and the velocity of the wheel go to zero.

controller	Settling time			Control force
	pendulum position	pendulum velocity	wheel velocity	
SFB controller	3sec.	3sec	4sec	0.01266
Fuzzy controller	1.3sec.	1.3sec	1.3sec	0.02815

Table 6.1: Control of Reaction wheel Pendulum: Comparison

Table 6.1 shows a comparative performance between state feedback controller and fuzzy logic controller (FLC) in terms of settling time and control input. It seen from this table that for state feedback (SFB) controller, settling times of pendulum position, pendulum velocity and wheel velocity are 3 seconds, 3 seconds and 4 seconds respectively. For fuzzy logic controller (FLC), settling time of position of pendulum, velocity of pendulum and velocity of wheel are 1.3 seconds, 1.3 seconds and 1.3 seconds respectively. The root mean square (RMS) value of control torque is 0.01266 Nm for SFB controller and 0.012815 Nm for FL controller.

6.5 Summary

The system dynamics of reaction wheel pendulum is derived using Euler-Lagrange formulation. It is a nonlinear dynamical system. A classical controller based on linearization technique and a fuzzy logic controller have been implemented to balance the reaction wheel pendulum in the vertical upward position. Only balancing control algorithm is implemented. The balancing is obtained with a convergence region of $\pm 10\ ^0$ (degrees) from vertical upward position. Fuzzy logic controller optimized with genetic algorithm, gives a very good performance compared with SFB controller in terms of settling time for same resign of attraction.

Chapter 7

Conclusion and Future Work

7.1 Conclusion

The real world system is highly complex and nonlinear in dynamics. Studies in nonlinear systems thus fascinated many researchers. It is difficult to obtain the exact dynamics of the nonlinear system as well as to design a suitable controller once the exact dynamical model is known. Unmodeled dynamics, parameter uncertainities are some of the characteristics which make the controller design more complex for nonlinear systems.

Concept of fuzzy set has been proposed by Zadeh in 1965 [23] to incorporate the human nature of understanding. Using the concept of fuzzy set, a fuzzy logic controller (FLC) can be designed based on some available informations about the system to be controlled. No exact mathematical model is required to design the control action of an FLC whereas the conventional control schemes depend on the exact dynamics of a system. Furthermore, FLC has a capability to tolerate uncertainties in the system which makes it more robust compared to conventional control schemes. Thus it is

worthwhile to implement FLC for nonlinear systems along-with conventional control strategies to see the advantages of it.

In this work, a number of nonlinear systems have been studied and various control techniques including FLC have been implemented on those systems. The nonlinear systems studied in this work include single link manipulator, reaction wheel pendulum and steam generator. The performance of FLC has been compared with those of conventional controllers for the above mentioned nonlinear systems. The fuzzy parameters have been optimized with genetic algorithm. In trajectory tracking of single link manipulator the performance of FLC is better than PD controller and as compatible with controller using feedback linearization. In balancing control of a reaction wheel pendulum, the performance of fuzzy logic controller is better than state feedback controller in terms of settling time.

The U tube steam generator has a nonlinear dynamics. It is required to control the water level of U tube steam generator for maintaining a dis-interrupted power supply.

A fuzzy logic controller has been designed for water level control of U tube steam generator in a nuclear power plant using the linearized model of the plant. Its performance is compared with state feedback controller designed for the same. It is seen that the performance of state feedback controller is better than fuzzy logic controller in terms of settling time.

For U tube steam generator, a dynamic model is necessary to analyze the performance of the controller. The design model should be simple enough to work with but must retain the essential features of the process. Detailed modeling of a U tube steam generator has been carried out based on the technical data provided by BARC. Some approximations have been made during modeling of the process. Using this model an

FLC, a PI and a PD controller is designed. It is seen that FLC gives better performance in terms of steady state error and settling time.

The above discussion concludes that FLC can be chosen as a better alternative for designing the controller of a nonlinear dynamical system when the exact dynamics is not known. Although it gives very good result there is no specific procedure for choosing the rule, the membership function etc. If this drawback can be taken into account, FLC can be widely used for many industrial applications in near future.

7.2 Scope for Future Work

- The proposed fuzzy logic controller is to be tested on an actual steam generator, which would be my endeavor once I resume my responsibility at BARC.
- A new FLC based on Lyapunov stability concept has been reported in the literature. It can be implemented on the boiler model described here and a performance comparison can be made.
- Many approximations have been made while modeling various components of the boiler. It would be interesting if these uncertain and unknown dynamics can be modeled using neural networks.
- While using a linear model for boiler in chapter 3, robustness aspect has not been taken into consideration. Since, linear approximation may not be quite close to the original system, it would be worthwhile to analyze the robust stability of proposed controllers.

***

Appendix

Linear Quadratic Regulator

Consider a linear system modeled by

$$\dot{x} = Ax + Bu \ , \ x(0) = x_0$$

with a performance index

$$J = \int_0^{\infty} \left(x^T Q x + u^T R u\right) dt$$

where $Q = Q^T \geq 0$ and $R = R^T \geq 0$

Find $u = -kx$ such that J is minimized.

If u is an optimum controller, then $\dot{x} = (A - Bk)x$ is also asymptotically stable.

$\Longrightarrow$ If $V = x^T P x$ then $\frac{dv}{dt}$ along the trajectory of the closed loop system is negative definite. Can one design an optimal control law for which $\frac{dv}{dt} \leq -(x^T Q x + u^T R u)$? This is proved in the following theorem.

Theorem

If $u^* = -kx$ such that

$$min_u \left(\frac{dv}{dt} + x^T Q x + u^T R u\right) = 0$$

for some $V = x^T P x$, then the controller is optimal.

Proof :

$$\left.\frac{dv}{dt}\right|_{u=u^*} + x^TQx + u^{*T}Ru^* = 0$$

$$\left.\frac{dv}{dt}\right|_{u=u^*} = -x^TQx - u^{*T}Ru^*$$

Integrating

$$V(x(\infty)) - V(x(0)) = -\int_0^\infty \left(x^TQx + u^{*T}Ru^*\right) dt$$

Since the closed loop system is stable,

$$\begin{aligned} x(\infty) &= 0 \\ V(x(0)) &= x_0^TPx_0 = \int_0^\infty \left(x^TQx + u^{*T}Ru^*\right) dt \\ \Longrightarrow J(u^*) &= x_0^TPx_0 \end{aligned}$$

To show that such a controller is optimal, here is a proof by contradiction. Let's assume that

$$min_u \left(\frac{dv}{dt} + x^TQx + u^TRu\right) = 0$$

at $u = u^*$, but u^* is not optimal. $\Longrightarrow$ There exists some $\tilde{u}$ such that $J(\tilde{u}) < J(u^*)$

$$\left.\frac{dv}{dt}\right|_{u=\tilde{u}} + x^TQx + \tilde{u}^TR\tilde{u} \geq 0$$

$$\Longrightarrow \left.\frac{dv}{dt}\right|_{u=\tilde{u}} \geq -x^TQx - \tilde{u}^TR\tilde{u}$$

Integrating

$$\begin{array}{ccc} V\left(x(0)\right) & \leq & \int_0^\infty \left(x^T Q x + \tilde{u}^T R \tilde{u}\right) dt \\ \uparrow & & \uparrow \\ J(u^*) & \leq & J(\tilde{u}) \end{array}$$

which is a contradiction.

<u>What is the optimal control law?</u>

$$f(u) = \frac{dv}{dt} + x^T Q x + u^T R u$$

At the minimum value where $u = u^*$

$$\frac{\partial f(u)}{\partial u} = 0 \text{ and } \frac{\partial^2 f(u)}{\partial u^2} > 0$$

Thus $\frac{\partial}{\partial u}\left(\frac{dv}{dt} + x^T Q x + u^T R u\right)\Big|_{u=u^*} = 0^T$

$$\begin{aligned} &\frac{\partial}{\partial u}\left(2x^T P \dot{x} + x^T Q x + u^T R u\right) \\ &= \frac{\partial}{\partial u}\left(2x^T P A x + 2x^T P B u + x^T Q x + u^T R u\right) \\ &= 2x^T P B + 2u^T R = 0^T \end{aligned}$$

This implies

$$\begin{aligned} u^T R &= -x^T PB \\ u^T &= -x^T PBR^{-1} \\ u &= -R^{-1}B^T Px \Longrightarrow k = R^{-1}B^T P \end{aligned}$$

Please note that

$$\begin{aligned} &\frac{\partial^2}{\partial u^2}\left(\frac{dv}{dt} + x^T Qx + u^T Ru\right) \\ &= \frac{\partial}{\partial u}\left(2x^T PB + 2u^T R\right) \\ &= 2R > 0 \end{aligned}$$

The optimal closed loop system has the form

$$\dot{x} = (A - BR^{-1}B^T P)x$$

However the matrix P is unknown which must satisfy the identity given in Theorem 7.2. Thus follows the derivation for P.

$$\begin{aligned} \left.\frac{dv}{dt}\right|_{u=u^*} + x^T Qx + u^{*T} Ru^* &= 0 \\ 2x^T PAx + 2x^T PBu^* + x^T Qx + u^{*T} Ru^* &= 0 \\ x^T(A^T P + PA)x - 2x^T PBR^{-1}B^T Px + x^T Qx + x^T PBR^{-1}B^T Px &= 0 \\ x^T(A^T P + PA + Q - PBR^{-1}B^T P)x &= 0 \\ \Longrightarrow A^T P + PA + Q - PBR^{-1}B^T P &= 0 \end{aligned}$$

This is referred to as the algebraic Riccati equation (ARE).

Please note that the control law derived here for which P is computed using ARE is valid only when control input is unconstrained.

linear State Feedback Control

It is a control technique for designing a controller for a plant with linearized model.

Consider the state space model of a system

$$\dot{x} = Ax + Bu \tag{7.1}$$

The control problem can be defined as, design a state feedback gain matrix K such that the control law given by

$$u = -Kx \tag{7.2}$$

where $K \in R^{m \times n}$ is a constant matrix. Then the closed loop system is

$$\dot{x} = (A - BK)x \tag{7.3}$$

The poles of the closed loop system are the roots of the characteristic equation

$$det(sI_n - A + BK) = 0 \tag{7.4}$$

We assume that the the designer made the selection of the desired poles of the close loop system, and they are

$s_1, s_2, s_3 \ldots\ldots\ldots\ldots s_n$

The desired closed loop pole can be real and complex. If they are complex they must come in complex conjugate pairs. We can form the desired close loop characteristic polynomial,

$$\begin{aligned} \alpha_c(s) &= (s - s_1)(s - s_2)(s - s_3)........(s - s_n) \\ &= s^n + \alpha_{n-1}s^{n-1} + ... + \alpha_1 s + \alpha_0 \end{aligned} \tag{7.5}$$

Our goal is to select the a feed back matrix K such that

$$det(sI_n - A + BK) = s^n + \alpha_{n-1}s^{n-1} + ... + \alpha_1 s + \alpha_0 \tag{7.6}$$

The above problem is called the pole placement problem. We first discuss the pole placement problem for the single input plants. In this case, $K = k \in R^{1 \times n}$. The solution to the problem is easily obtained if the pair (A, b) is already in the controller companion form. In such case we have

$$A - bk = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ & & & & & \\ 0 & 0 & 0 & \dots & 0 & 1 \\ -a_0 - k_1 & -a_1 - k_2 & -a_2 - k_3 & \dots & -a_{n-2} - k_{n-1} & -a_{n-1} - k_n \end{bmatrix}$$

Hence the desired gains are

$$k1 = \alpha_0 - a_0$$

$$k2 = \alpha_1 - a_1$$

$$.$$

$$k_n = \alpha_{n-1} - a_{n-1}$$

If the pair (A, b) is not controller companion form, we first transform into companion form and then compute the gain vector $\tilde{k}$ such that

$$det(sI_n - \tilde{A} + \tilde{b}\tilde{k}) = s^n + \alpha_{n-1}s^{n-1} + ... + \alpha_1 s + \alpha_0 \tag{7.7}$$

thus,

$\tilde{k}$=$[\alpha_0 - a_0,\ \alpha_1 - a_1,.........\ \alpha_{n-1} - a_{n-1}]$

Then,

$k = \tilde{k}T$,

Where T is the transformation that bring the pair (A, b) into controllable companion form. It can be represent the above formula

$$\tilde{k}T = [\alpha_0 - a_0, \alpha_1 - a_1, ...\alpha_{n-1} - a_{n-1}][q_1, q_1A.....q_1A^{n-1}]^T \tag{7.8}$$

$$= q_1(\alpha_0 I_n + \alpha_1 A + ... + \alpha_{n-1}A^{n-1}) - q_1(a_0 I_n + a_1 A + ... + a_{n-1}A^{n-1})$$

By the Cayley-Hamilton theorem, we have

$A = -(a_0 I_n + a_1 A + ... + a_{n-1}A^{n-1})$

Hence,

$k = q_1 \alpha_c(A)$

The above expression for the gain row vector was proposed by Ackerman in 1972.

Feedback Linearization

The central idea of feedback linearization is to transform a nonlinear system dynamics into a linear one using (dynamic) feedback. The advantage is evident that once the system is transformed into linear one, all linear control techniques can be applied. The theory of linearization is developed from Differential Geometry[24]. There are basically two kinds of approaches for linearization, namely

1. Input-Output linearization.

2. Input-State linearization.

Input-Output Linearization

Consider a single input single output (SISO) nonlinear system

$$\dot{x} = f(x) + g(x)u \tag{7.9}$$

$$y = h(x) \tag{7.10}$$

See in the above equation (7.9) output is related to input through state. The objective is to bring out direct linear relationship between input and output

$$y^m = v \tag{7.11}$$

where $v = \gamma(x, u)$,

$m \in R$ indicates differentiation by m times and $m \leq n$ and m is known the relative degree of the system(7.9).

If in the case $m < n$, then the transfered system (7.11) will have unobservable dynamics of order $n - m$. MIMO systems are also dealt in the same way with bit more complexity. This technique has been successfully implemented in several practical applications, such as flight control [25] and the control of rigid robots by so-called computed torque method [26].

Input-State Linearizion

In this approach, the objective of linearizion is achieved in two steps, State transformation and Input transformation. Moreover in this approach we are not concerned with output. The formula definition of the problem (in SISO case) is given in [27]. Consider the system

$$\dot{x} = f(x) + g(x)u \tag{7.12}$$

the objective is to find out a state and input transformations

$$v = q(x) + s(x)u \tag{7.13}$$

$$z = T(x) \tag{7.14}$$

so that the original system is transformed into linear one.

$$\dot{z} = Az + Bv \tag{7.15}$$

In fact in the equation (7.11), relative degree, m = n, order of the system (7.9) then Input-Output linearization leads to Input-State Linearization with state transformed from $x \longrightarrow y$.

Control design using feedback linearization involves partial differential equations and solving these equations is really difficult particularly in the case of MIMO systems. Still the topic is under study and the research is underway to overcome the problem of solving partial differential equations.

Under-actuated Mechanical Systems

Under actuated systems are those systems of the form

$$\ddot{q} = f(q, \dot{q}) + G(q)u \tag{7.16}$$

where,

q is the state vector of independent generalized coordinates, (also called as vector of configuration variables).

f(.) is the vector field representing the dynamics of the systems.

$\dot{q}$ is the generalized velocity vector.

G is the input matrix.

and u is a vector of generalized force inputs.

The dimension of q is defined as the degrees of freedom of the system . In general (rather literally) a system is said to be under actuated if the external generalized forces are not able to command instantaneous accelerations in all directions in the configuration space, i.e. $rank(G) < dim(q)$.

Homogeneous Density

As described in equation (4.10) the homogeneous density of water and steam is given as

$$D_{Uh}(t) = \frac{1}{S_{qp}/(2.0 \times D_{ss}(t)) + (1 - S_{qp}/2.0)/D_{sw}(t)} \tag{7.17}$$

These parameters are described in section 4.5.1

Proof of equation (4.10)

$$\frac{1}{d1_{Uh}} \tag{7.18}$$

$$\Rightarrow \frac{V}{m_f} \tag{7.19}$$

$$\Rightarrow \frac{1}{m_f} \times (V_s + V_w) \tag{7.20}$$

$$\Rightarrow \frac{1}{m_f} \times (m_s/d1_{ss} + m_w/d1_{sw}) \tag{7.21}$$

$$\Rightarrow \frac{1}{m_f}(m_f \times (St_{qp}/2)/d1_{ss} + m_f/d1_{sw}) \tag{7.22}$$

$$\Rightarrow \frac{m_f}{m_f} \times (St_{qp}/(2.0 \times d1_{ss}) + (1 - St_{qp}/2.0)/d1_{sw}) \tag{7.23}$$

$$\Rightarrow St_{qp}/(2.0 \times d_{ss}(t-1)) + (1 - St_{qp}/2.0)/d_{sw}(t-1) \tag{7.24}$$

From equation (4.10) and (7.24), it proves the formula used for calculation the homogeneous density. The parameters used from boiling section to U tube resign are

V= Volume of steam generator.

m_f= Mass of flow(Kg).

m_s= Mass of steam flow (Kg).

m_w= Mass of water flow (Kg).

V_s= Volume occupied by steam (Kg).

V_w= Volume occupied by water $meter^3$.

Terms related to modeling of the process

Saturation temperature: The temperature at which a liquid substance boils under a given pressure.

Latent heat of vaporization: The heat that changes the physical state of a substance from a liquid to a vapor, or from a vapor to a liquid; no temperature change is shown by a thermometer during the conversion process. It is a function of pressure and temperature.

Specific heat: The quantity of heat require to raise the temperature of a substance of unit mass to unit degree under a given pressure and temperature.

Saturated steam: Steam at the temperature and pressure at which evaporation occurs.

unsaturated steam: when both the water and liquid are present steam and water mixture, then the steam is called unsaturated steam. **Saturated water:** Water at its boiling point.

Steam quality: The percent by weight of vapor in a steam and water mixture.

Density : It is the ratio of mass to volume of a substance.

Homogeneous density : When the density is calculated for the combination of two substance it is called homogeneous density. as described in section 4.5.1, the homogeneous density for water and steam is calculated.

Bibliography

[1] A. Roulet P. Ruiz Framatome A. Gautier, J. F. Petotrot and G. Zwingelstenin. U-tube steam generator modelling: application to level control and comparision with plant data. *Boiler dyanamics and control in nuclear power station. BENS, Lomdon*, 1979.

[2] A.M. Hasanul Basher and J. March Leuba. Development of a robust model-based water level controller for a u-tube steam generator. *U.S. department of Energy*, September 2001.

[3] H. Levent Akin and Vural Altin. Rule-based fuzzy logic controller for a pwr-type nuclear power plant. *IEEE Trannsactions on Nuclear Science*, 38(2):883–890, 1991.

[4] Byuog Hak Cho and Hee Cheon No. Design of stability guaranteed fuzzy logic controller for nuclear steam generator. *IEEE Transaction on Nuclear Science*, 43(2):717–730, April 1996.

[5] Mehrdad Saif. Suboptimal projective control of a pressurized water reactor. *IEEE Tranjection on Nuclear Science*, 36(6):2459–2465, December 1989.

[6] J.J. Slotine and S.S. Sastry. Tracking control of non-linear systems using sliding surfaces with application to robot man. *LIDS*, 1982.

[7] R SPURGEON, S K; DAVIES. A nonlinear control strategy for robust sliding mode performance in the presence of unmatched uncertainty. *International Journal of Control*, 57(5):1107–1123, may 1993.

[8] B De Luca, A; Siciliano. Trajectory control of a non-linear one-link flexible arm. *International Journal of Control*, 50:1699–1715, 1989.

[9] Sheldon L. Trubatch Riza C. Berkan. *Fuzzy Systems Design Principles.* IEEE PRESS, first edition, 2000.

[10] Jeffrey B. Burl. *Linear Optimal Control.* ADDISON-WESLEY, 1999.

[11] Stanislaw h. zak. *Systems and control.* Oxford University Press, first edition, 2003.

[12] Bonagiri Bapira ju. Nonlinear control strategies for underactuated mechanical systems. Master's thesis, IIT Kanpur, 2004.

[14] Qing Lu. Design of fuzzy controller for a smart structures using genetic algorithms. *Smart Materials and Structures*, 2003.

[15] P K Nag. *Power Plant Engineering.* Tata McGraw-Hill publicing Company Limited, 1999.

[16] Erik S. Pedersen. *Nuclear Power Plant Design*, volume 1. ANN ARBOR SCIENCE, 1978.

[17] A.S. Sarao. *Thermal Engineering.* satya prakashan, 1995.

[18] Jay Farrell W. Li, X. G. Chang and F. M. Wahl. Design of an enhanced hybrid fuzzy p id controller for a mechanical manipulator. *IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS PART B: CYBERNETICS*, 2001.

[19] Weping Li Jean-jacques E. Stotine. *Applied Nonlinear control.* Prentice Hall Englewood Cliffs, New Jersey 07632, 1991.

[20] Mark W. Spong. Nonlinear control of the inertia wheel pendulum. *Automatica*, 37, 2005.

[21] M. Gopal I.J. Nagrath. *Control System Enginnering.* New Age International Publishers, third edition, 1999.

[22] Kim Chwee Ng and Yun Li. 'design of sophisticated fuzzy logic controllers using genetic algorithms. *Proc. 3rd IEEE Int. Conf. On Fuzzy Systems, Orlando, FL*, 1994.

[23] L. A. Zadeh. Fuzzy sets. *Information and Control*, 8:338–353, 1965.

[24] Alberto Isidori. *Nonlinear Control Systems.* Springer-Verlag London, 1995.

[25] S. H. Lane and R. F. Stengel. Flight control design using nonlinear inverse dynamics. *Automatica,*, 24:471483, 1988.

[26] E. Freund. The structure of decoupled nonlinear systems. *Int. J. of Control*, 1975.

[27] M. Vidyasagar. *Nonlinear Systems Analysis.* Prentice Hall, 1993.